BLACK & HATCH

Transformer Characteristics

Electrical Engineering

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TRANSFORMER CHARACTERISTICS

BY

CHARLES DAY BLACK
RALPH SNYDER HATCH

THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE
IN
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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Charles Day Black and Ralph Snyder Hatch

ENTITLED Transformer Characteristics

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Electrical Engineering

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TRANSFORMER CHARACTERISTICS.

INTRODUCTION.

The object of this investigation was to determine the various characteristics of the new type H , form K , General Electric transformer, when operated on both single and three-phase circuits under various conditions. The magnetic circuit of the transformer, which because of its shape might be called either a core or shell type, is made of a new grade of silicon steel having such physical qualities that the core loss is small.

All single phase measurements were made on the same transformer, and in the three-phase tests the two additional transformers used were of exactly the same size and type. In all the tests made emphasis was laid upon accuracy, and it is believed that within reasonable limits such was obtained.

Schedule of Transformer Data.

Data obtained from the Transformer.

Rating:

No. 775063 .

Type H - Form K - 60 cycles.

5 K.W. Volts Pri. 1100/2200 Volts Sec. 110/220 .

Features of construction:

Oil cooled.

Fastened to case by set-screws. About one inch space between transformer and case.

Magnetic circuit:

Four magnetic circuits in parallel as shown in figures 1 and 2 . Thickness of laminations = 0.0416 in. (approx.)

Volume of core:

V_1 = vol. of center leg = 907 cu.cm. = 55.35 cu.in.
V_2 = " " outside legs = 1358 " " = 82.9 " "
V_3 = " " ends = 2600 " " = 158.75 " "
V_T = net volume of core 4865 " " = 297.0 " "

Weight of core (0.017 lbs. per cu.cm.) = 83 lbs.(approx.)
" " " per K.W. capacity = 16.6 lbs.

Mean length of magnetic path = 24 inches = 61 centimeters.

Winding data:

Average length of primary turn = 21.5 inches.
" " " secondary " = 21.5 " .
Total length of wire (primary) = 2400. feet.
" " " " (secondary) = 240. " .
" weight " " (primary) = 23.5 lbs.
" " " " (secondary) = 18.2 " .

Diagram of magnetic circuit:

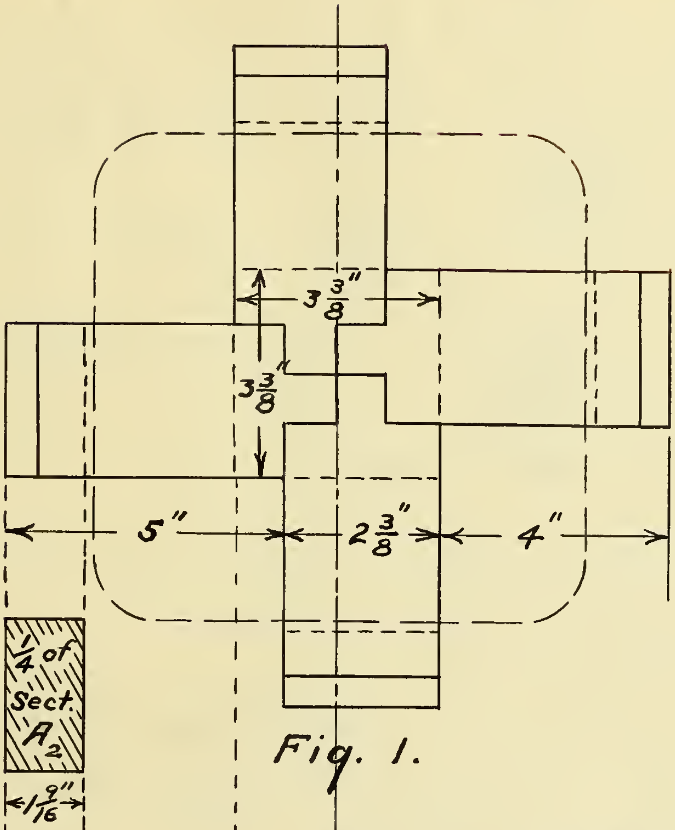

3 3/8"
3 3/8"
5"
2 3/8"
4"
1/4 of Sect. A2
1 9/16"

Fig. 1.

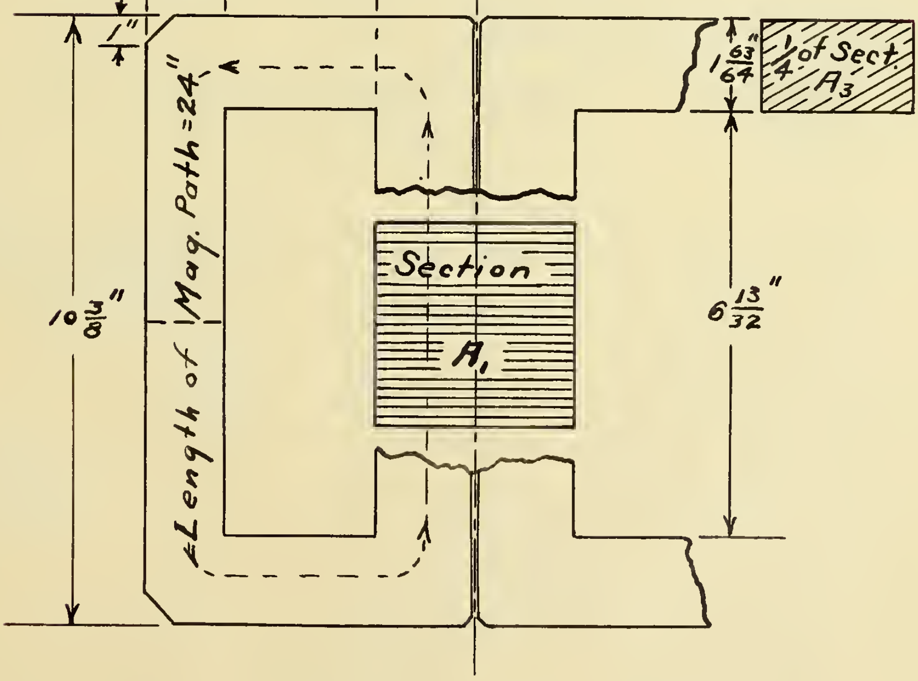

1"
Length of Mag. Path = 24"
10 3/8"
1 63/64"
1/4 of Sect. A3
Section A1
6 13/32"

Fig. 2.

Resistance of coils:

	Ohms at 18.6°C	Ohms at 25°C	Ohms at 50°C
Low voltage coil No. 1 .	0.04858		
" " " " 2 .	.04853		
" " coils in series.	.09711	0.09945	0.1089
High voltage coil No.1 .	3.9475		
" " " " 2 .	3.9388		
" " coils in series.	7.8863	8.083	8.851

Full load current for primary = 2.27 amperes.

" " " " secondary = 22.7 " .

Data obtained from Manufacturers:

No. of turns per coil of primary = 660.

" " " " " " secondary = 66.

Total number of primary turns = 1320.

" " " secondary " " 132.

Cross-sectional area of center leg = 8.64 sq.in. = A_1 .

" " " " outside legs = 12.93 " " = A_2 .

" " " " ends = 16.17 " " = A_3 .

Tests.

It seemed advisable in the first place to determine the wave shape of the e.m.f. impressed on the transformer under extreme as well as normal conditions, since it is customary to study and discuss the losses of the transformer as they exist when a sine wave of e.m.f. is impressed. To do this oscillographs were taken and some of the waves analized thus making it possible to determine the effect of pulsating armature reaction on the wave form when the machine was operated as a single phase alternator.

Reactance test:

The reactance and copper loss test was made in the usual manner, that is, by measuring the power input to the high voltage side with the low voltage coils short-circuited. Readings of current and voltage impressed were also taken, carrying the amount of current to about double the full-load value. (Since the flux was negligible, all power measured was I^2R loss).

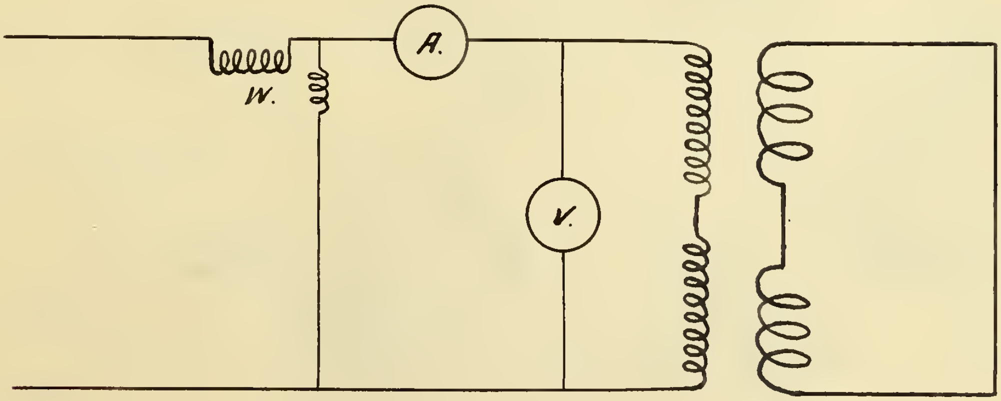


Fig. 3.

Single phase core loss test:

The diagram below will show the method of balancing the load on the alternator in the single phase core loss tests. Data was taken at 30 and at 60 cycles, and in addition tests were carried thru for both constant excitation and constant impressed e.m.f.

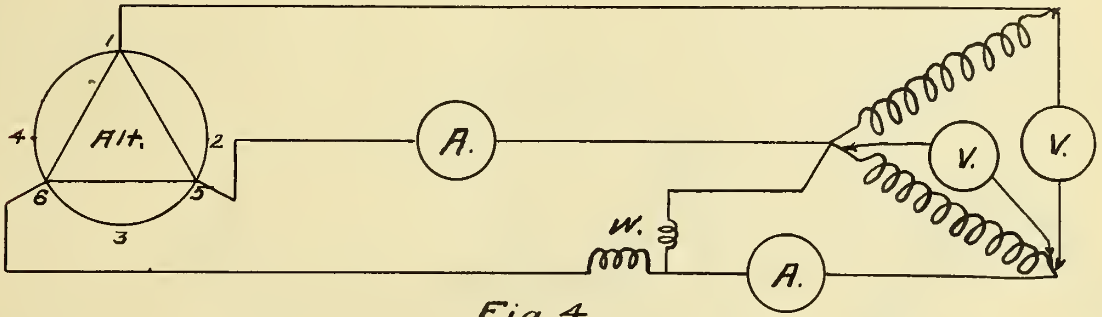


Fig. 4.

Core loss of an individual transformer in a three-phase system:

By measuring the loss in the separate transformer, when connected in the three ways shown below, all the typical conditions are represented. In other words, these connections involve all the important feasible methods.

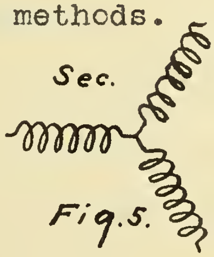
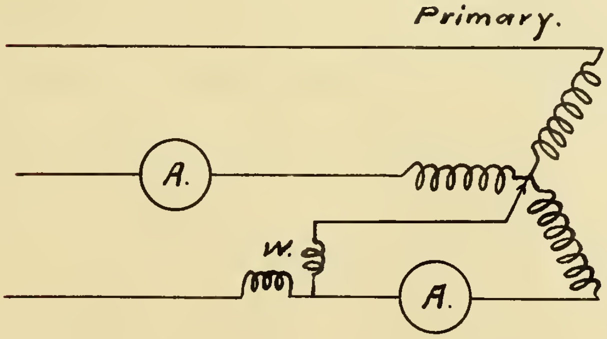


Fig. 5.



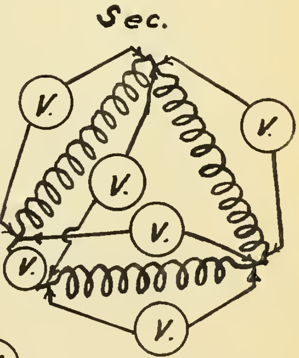


Fig. 6.

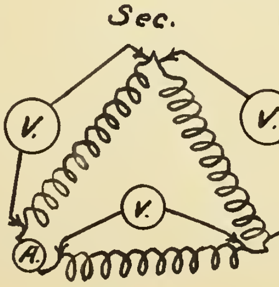


Fig. 7.

Core loss of three transformers by the two-wattmeter method:

In addition to the above tests, another was made, two wattmeters being used to measure the combined core losses of the three transformers at constant frequency with the closed delta secondary. By dividing the total watts consumed by three the average loss per single transformer was found for different voltages.

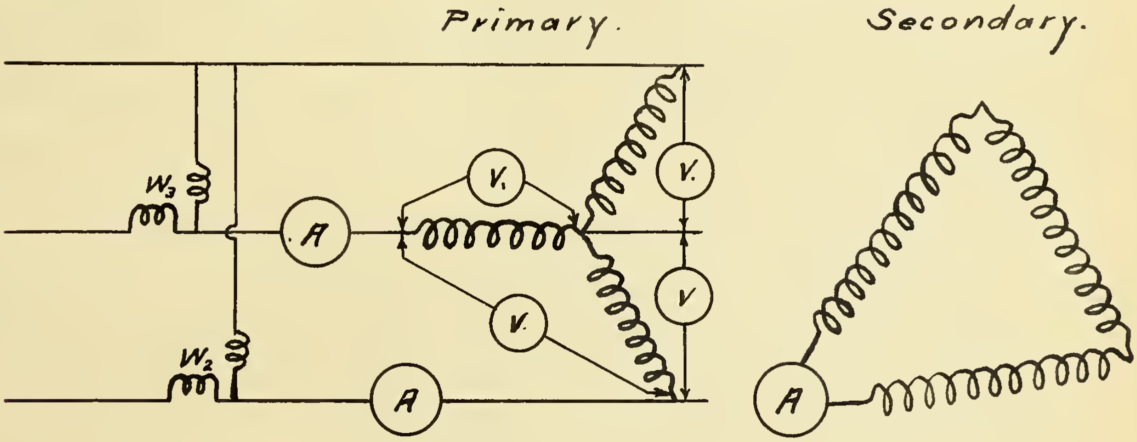


Fig. 8.

Determination of the hysteresis loop:

Data for the general hysteresis loops at normal excitation, over-excitation, and with full-load current as exciting current was obtained by Rowland's ballistic method. These loops have been plotted to the same scale and appear in the last pages of this thesis.

Investigation of E.M.F. Wave of Generator.

Since it was necessary to have some source of e.m.f. that could be adjusted to the desired conditions under which the tests were to be made, and since a machine of proper size was not available, a 7 1/2 K.W., 4-pole, rotary converter designed for 500 volts D.C. was used, this being the only machine available. Originally the field pole windings were in series; but in order to give the proper flux the winding was changed, leaving two poles only in series, thus making it possible to excite the field with 220 volts D.C. from an external source. Full-load energy current for this machine running as an A.C. generator, (D.C. brushes off the commutator), connected 3-phase and generating 220 volts, would be 19.7 amperes. The transformers to be tested were 5 K.W. and with the two low voltage coils in series required 22.72 amperes at full load.

In order to supply full-load current for the transformer as exciting current in the core loss tests, over 300 volts were required, and this much current was a considerable overload for the generator. In order to investigate the e.m.f. wave with this overload on one phase only, oscillographs were taken of the e.m.f. wave at 411 volts no-load, and at 411 volts with a load of 23 amperes; and the e.m.f. waves analized by Fourier's series for fundamental, third and fifth harmonics. This was done in order to determine how badly the waves were distorted by the pulsating armature reaction which existed as a result of the heavy overload on only one phase of the machine. The computations involved in the analysis are not

given but the waves with their components are shown on page 86 .

The triple frequency component in the distorted wave of 411 volts was comparatively large; hence, as the distortion was considerable, it was decided that in making the core loss tests the three phases should be loaded about equally in order to maintain as nearly a sine wave of e.m.f. as possible. This was accomplished by using two transformers, connected as in Fig. 4 , consuming 3-phase power.

Hysteresis Loop and Losses.

In the study of the transformer it is desirable to have the saturation curve of the iron used, as well as the hysteresis loop at normal density at least, and if possible at other densities. As the dimensions and sketch of the transformer core show, the magnetic circuit has three different cross-sections of iron in series. It is needless to say that a path of this shape complicates all calculations that have to be made in the determination of the value of the permeability, μ , Steinmetz' coefficient, η , and exponent, x , which are used in separating the losses.

A magnetization curve for the iron was not available and because of the limited time allowed for making tests and preparing this thesis, and also because of the difficulties encountered in carrying out satisfactory tests on samples of the transformer iron, it was not possible to obtain a curve from which the permeability could be calculated. True hysteresis loops at desired densities were not obtained in a satisfactory way, but the method of carrying out such tests, and also the theory involved, is taken up in another part of this discussion. However, the general magnetization curve plotted between volts and amperes, is shown on page 96 . The data for this curve was taken from the results obtained in the coreloss test at normal frequency, made using the V-connection for balanced load on the alternator.

As stated above under "Tests", general hysteresis loops were obtained, which represent what might be called an average

flux density because of the three different cross-sectional areas offered as a path for the flux. Since the loss due to hysteresis plays such an important part in determining the heating, and consequently the efficiency of the transformer, it is well to know the constants that are necessary to compute this loss. Several methods may be used to get the hysteresis loss which is represented by the well known equation:

$$P_h = \eta V f \beta^x (10)^{-7}$$

where,

P_h = loss in watts due to hysteresis,

V = volume of iron in cu. cm.

η = Steinmetz' coefficient which depends upon the quality of the iron, its physical condition,etc.

X = Steinmetz' exponent which is approximately 1.6 for most iron and densities ordinarily employed.

β = maximum value of the flux density in lines per sq. cm.

In regard to the relation of the hysteresis loop and the loss due to molecular friction, a word of explanation may be advantageous. When an alternating flux is produced in iron or other magnetic materal, a loss of energy takes place because of a kind of frictional resistance between the molecules of the substance. This phenomenon just described is

called molecular magnetic friction. If these alternations are caused by an alternating current and no other energy is expended upon the magnetic circuit by vibration, jar, or other means, the real energy consumed by molecular magnetic friction has to be supplied by the alternating current. If external work is done on the magnetic circuit, for example, by vibration, and the hysteresis loop is taken at the same time, the loop collapses to a greater or less degree and its area no longer holds any definate relation to the actual energy consumed by molecular magnetic friction, or hysteresis.

The ballistic method mentioned above is the standard one for testing hysteresis loss, tho more rapid commercial methods are now in use. The solution, by the use of the hysteresis loop, for the constant η in the formula for hysteresis given above will be taken up following the determination of the permeability μ.

(a) Determination of Permeability μ.

Two methods of determining the permeability μ are described in the following. Inherent difficulties prevented accurate results from being obtained with either of them, nevertheless the methods pursued will be given. The test made using the magnetometric method was incomplete in that with the available solenoids a high enough value of field could not be obtained. (β = 5,000 lines per sq.in. was the highest value reached.) Time for winding a special solenoid was not available. Also, since none but short

lengths of the sample could be obtained from the transformer, the value of the ratio of length to breadth could not be made large enough for accurate work without making the specimen very narrow.

First method:

The determination of the permeability, μ , may be made from the saturation curve if the cross-section of the magnetic path is known. This method, however, is not very accurate because the flux density is not uniform thru the cross-section. The lines of force tend to shorten and hence the density is greatest on the inside of the magnetic circuit as shown in Fig. **9** .

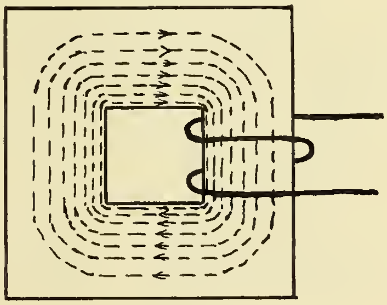


Fig. 9.

μ is defined as the ratio of the flux β per unit cross-section to the magnetizing force $\mathcal{H}$ per unit length. The magnetization or saturation curve is not a straight line, hence the value of $\frac{\beta}{\mathcal{H}}$, or μ varies. For this reason the value of μ will not be constant in different sections of the transformer magnetic circuit. How great an error is made by assuming the value of β constant throughout the section depends on what part of the saturation curve the transformer is

operating. An accurate calculation of this error would be a tedious mathematical process and will not be taken up here, because it would have to be calculated for each value of flux used. Neglecting this error the method is probably accurate to within 10% .

In the transformer tested each magnetic path consisted of three separate parts, each of which had a different cross-sectional area. For this reason no calculation with any known degree of accuracy can be made to determine the value of μ . As an illustration only, of the method to be used, the problem will be carried thru using a mean value of cross-sectional area.

From the schedule of transformer data:

	Cross-sectional area.		Mean length of magnetic path.
Center leg	8.64 sq.in.	= 55.7 sq.cm.	16.5 cm.
Outside legs	12.93 " "	= 83.4 " "	16.5 ".
Ends	16.17 " "	= 104.3 " "	28.0 ".
	Total mean length of path	=	61.0 ".

Volume of path.

Center leg----- $55.7 \times 16.5 = 920$ cu. cm.

Outside legs--- $83.4 \times 16.5 = 1376$ " ".

Ends-----------$104.3 \times 28.0 = 2922$ " ".

Total volume of path = 5218 " ".

Mean cross-sectional area $= 5218 \div 61 = 85.6$ sq. cm.

From the fundamental transformer equation,

$$E = \frac{4.44\ f\ n\ \Phi}{(10)^8} = \frac{4.44\ f\ n\ A\ \beta_{max.}}{(10)^8} .$$

we obtain:

$$\beta_{max.} = \frac{E \times (10)^8}{4.44\ f\ n\ A}$$

where, E = effective A.C. voltage.
f = frequency.
n = number of turns thru which exciting current is sent.
A = cross-sectional area of magnetic circuit.
β = maximum value of flux density in whatever units A is taken.

hence,

$$\beta_{max.} = \frac{22,000,000,000}{4.44 \times 60 \times 132 \times 85.6}$$

$$\beta_{max.} = 7,300 \text{ lines per sq. cm.}$$

From the regulation data, normal magnetizing current = .365 amp.

Number of secondary turns = 132 .

A.C. ampere-turns per centimeter length of magnetic circuit

$$= \frac{132 \times 0.365}{61} = 0.79$$

$$\mathcal{H} = \frac{4\,\pi\ n\ I}{10} = \text{magnetizing force per centimeter length}$$

where nI are the ampere turns, if I = direct current, or the maximum value of the A.C. current.

Therefore $nI = 0.79\sqrt{2} = 1.116$ A.T. is the value to substitute in the above formula, hence:

$$\mathcal{H} = \frac{4\,\pi \times 1.116}{10} = 1.402 \text{ Gilberts.}$$

From these results:

$$\mu = \frac{\beta}{\mathcal{H}} = \frac{7,300}{1.402} = 5,210 \ .$$

Since the curvature changes but little for a considerable range on either side of the operating point on the magnetization curve, the value of μ must remain nearly constant in the different sections.

Second method:

The magnetometric method **offers an** accurate means of determining the β and $\mathcal{H}$ curve of a specimen of the transformer iron, providing the proper conditions may be obtained. Among the more important of these conditions is the requirement that the place where the test is made be entiroly free from vibrating disturbances, and that there be no shifting magnetic fields in the vicinity. Even a bunch of keys or a knife carried in the pocket will affect the sensitive magnetometer needle. The test was made at night in one of the laboratories of the Physics Building so that the best possible conditions would be obtained. Even then it was noticed that the magnetometer was affected very slightly by the stray fields of the street-car motors a full block distant.

The apparatus used and its arrangement is shown in Fig. 10.

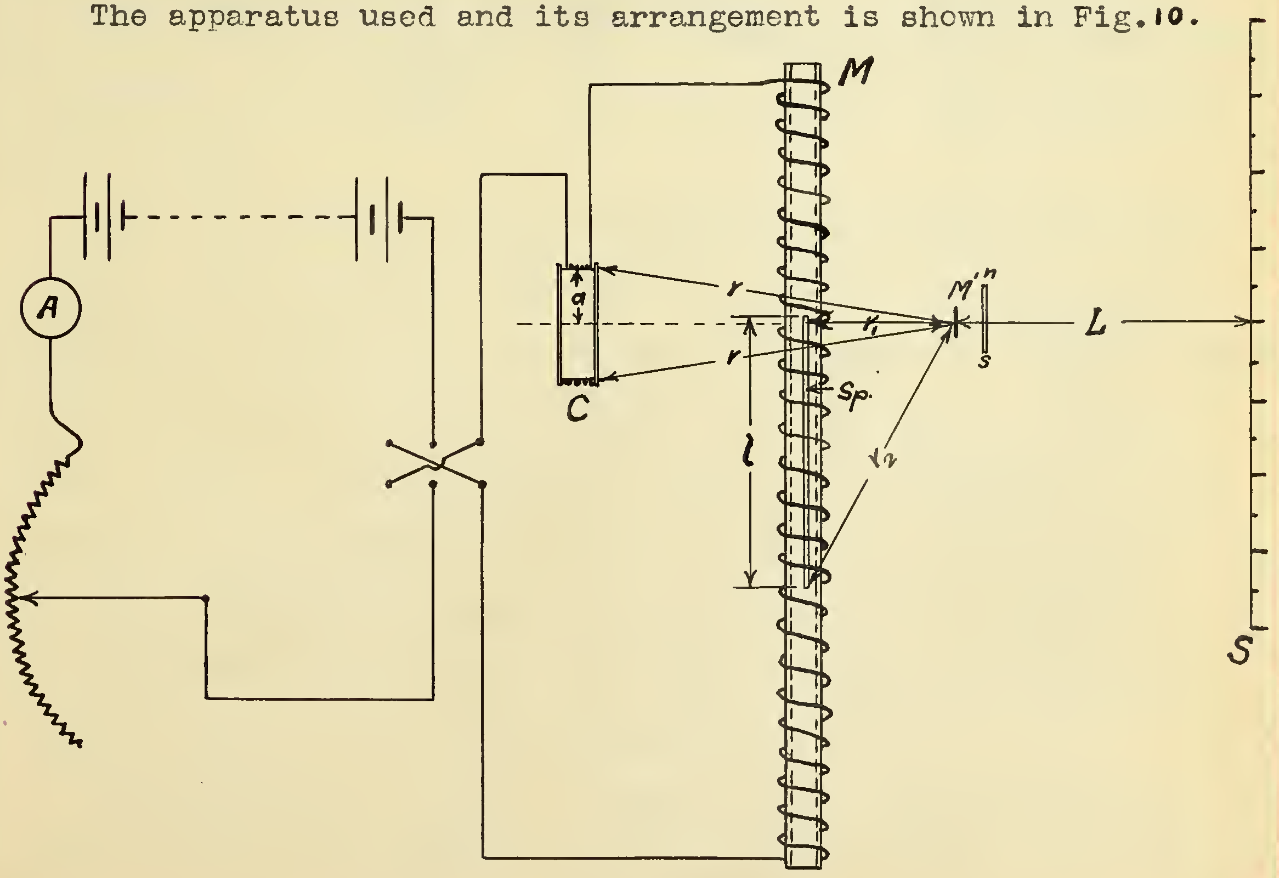


Fig. 10.

The general theory is as follows:

A pole of unit strength is one which repels an equal and like pole at a distance of one centimeter with a force of one dyne. Convention defines the density of the magnetic lines from this pole at a distance of one centimeter from its center as one line per sq. cm., hence there emanate from the unit pole 4π lines of force, and from the pole of strength m, $4\pi m$ lines. The strength of field is denoted by $\mathcal{H}$ and is numerically equal to the number of lines per sq. cm. in air. A turn of wire thru which a current is flowing produces a field, and that value of current which will produce a pole strength of unity is one unit of current in the c.g.s. system. It is equal to ten times the practical unit of current, the ampere, and is called the abampere. The value of field within the single turn is therefore $\frac{4\pi I}{10}$ where the current is expressed in amperes, and $\frac{4\pi nI}{10}$ when there are n turns per centimeter length.

In a true solenoid the poles produced by the field are near the end, and the distribution thru the middle portion of the solenoid is uniform, the field $\mathcal{H}$ being equal to $\frac{4\pi nI}{10}$ since all of the lines must pass thru the tube and out at the ends. The strength of pole formed will depend on the number of lines emanating from it and hence upon the area enclosed by the tube, but the strength per unit area will be proportional to the intensity, $\mathcal{I}$, of the pole, or to $\frac{m}{s}$, where m is the pole strength and s its cross-sectional area.

The permeability of air is taken as one. The introduction into the tube of a substance of greater permeability than air

will increase the flux density in that portion of the tube which this substance occupies by an amount equal to the original field times the increase of permeability over that of air, or $(\mu - 1)\mathcal{H}$ expresses the increase of density, and the total density will be given by $(\mu - 1)\mathcal{H} + \mathcal{H}$. This reduces to $\mu\mathcal{H} = \beta$, or $\mu = \frac{\beta}{\mathcal{H}}$, the defination of permeability.

The induced magnetism creates poles in the end of the iron specimen. If m = strength of one of these poles the induced magnetism is given by: $4\pi m$. Let s = cross-section of the iron specimen, then the total number of lines passing thru the iron equals $s\mathcal{H}$ due to the inducing field, and $4\pi m$ lines due to the induced magnetization. Therefore we have:

$$\beta = s\mathcal{H} + 4\pi m = \mathcal{H} + 4\pi\frac{m}{s}$$

$$= \mathcal{H} + 4\pi\mathcal{J}\text{, since as previously seen, } \mathcal{J} = \frac{m}{s}.$$

It now remains but to obtain some method of measuring the pole strength. This is what the magnetometer does by comparison, that is, the magnetometer is first calibrated for the horizontal component, $\mathcal{H}_e$, of the permanent field affecting the magnetometer needle during the test, and then this field is compared with the resultant field, at the magnetometer, of the two poles of the specimen.

The proceedure in the test is as follows: The specimen is suspended in the vertical solenoid in such a position that it has the maximum effect on the magnetometer with a given value of $\mathcal{H}$. It is then known that the pole due to the induced magnetization is in a horizontal line with the magnetometer mirror on the back of which is the magnetic needle. The length

of the specimen is taken as the distance between poles, assuming that the pole at the other end is similarly located. The permanent magnet, n-s , is now placed so that with the specimen removed and no current flowing the zero of the reflected lamp filament on the scale will not be changed, but so that when the specimen is replaced and the maximum current sent thru the coil the reading will still be on the scale.

With the specimen removed the position of the compensating coil, C, is adjusted with its plane perpendicular to the horizontal center line thru it, the mirror, and the scale zero, until with any value of current thru it and the magnetizing coil, M, in series, there will be absolutely no deflection of the needle; then the effect of any field in M on the needle will always be compensated for by the field produced by C . The coil M is now cut out of series and the current sent thru C alone to determine the value of $\mathcal{H}_e$.

To determine $\mathcal{H}_e$:

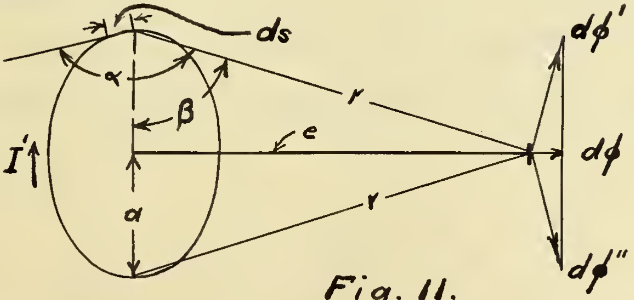


Fig. 11.

Suppose the current I' passing thru the compensating coil. The lines of force always cross the line r at right angles to it. Let the field at the magnetometer needle due to the current I' in the element ds be represented by the vector $d\phi'$, and that due to an equal element diametrically opposite by $d\phi''$. It is seen immediately that the components of these fields perpendicular to e are equal and opposite, while all the

components parallel to e are in the same direction and add up.

$$d\phi = \frac{n I' ds}{r^2} \cos\beta \qquad \cos\beta = \frac{a}{r}$$

a = diam. of coil.

$$= \frac{n I' ds\ a}{r^3} \quad .$$

n = no. of turns.

r = dist. from M' .

Integrating, $\phi = \frac{I' n a}{r^3} \int_0^{2\pi a} ds$

$$= \frac{2\pi a^2 n}{r^3} I' \text{ , for } I' \text{ in abamperes.}$$

$$= \frac{2\pi n a^2}{10 r^3} I' \text{ , for } I' \text{ in amperes.}$$

Since the tortion of the mirror's suspending fibre is adjusted so that it is nil when the needle lies parallel to the field, $\mathcal{H}_e$, when equilibrium is reached and θ is the angle thru which the needle is turned, the restoring torque will be proportional to sine θ and the displacing torque to cos θ , or :

$$\mathcal{H}_e\ l\ m' \sin\theta = \phi\ l\ m' \cos\theta$$

from which, $\mathcal{H}_e = \phi \cot\theta$

$$= \frac{2\pi a^2}{10 r^3} I' \cot\theta \quad .$$

The value of $\mathcal{H}_e$ is determined from several readings of I' , and the mean taken. Upon the accuracy with which $\mathcal{H}_e$ is determined depends to a large extent the accuracy of the deter-

mination of μ ; therefore, great care has to be taken in reading the deflection, and since the angles are small, in using the correct value of the cot θ . Deflections are taken both to right and left by reversing the current, and the average value is used.

Before taking the data for the β and $\mathcal{H}$ curve, the specimen is entirely demagnetized by means of an A.C. helix and water rheostat, or by putting it in the coil M and decreasing the current slowly from a large value to zero by small increments, at the same time continually reversing the current thru M . The specimen is near enough to being entirely demagnetized when it has no effect on the needle, that is, the needle remains on zero whether or not the specimen is removed.

The general relation of force, pole strength, and distance is:

$$F = \frac{m\,m'}{r^2} .$$

Force due to the pole at (a) = $F_1 = \frac{m}{r_1^2}$.

" " " " " " (b) = $F_2 = \frac{m}{r_2^2} \cos\alpha$.

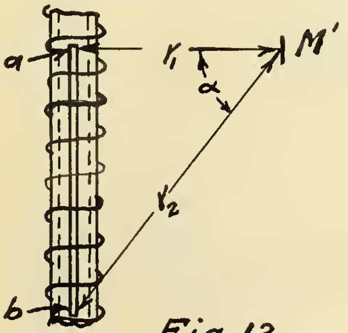


Fig. 12.

The resultant force along r ,

$$F = F_1 - F_2 = \frac{m}{r_1^2} - \frac{m}{r_2^2} \cos\alpha .$$

The two poles are equal and opposite.

Therefore,
$$F = m\left\{\frac{1}{r_1^2} - \left(\frac{1}{r_2^2} \times \frac{r_1}{r_2}\right)\right\}$$

$$= \frac{m}{r_1^2}\left\{1 - \left(\frac{r_1}{r_2}\right)^3\right\} ,$$

and substituting, $m = \mathcal{J} s$,

$$F = \frac{\mathcal{J} s}{r_1^2}\left\{1 - \left(\frac{r_1}{r_2}\right)^3\right\} .$$

For equilibrium the two turning moments of the external field are equal; hence, as seen from Fig. 13 ,

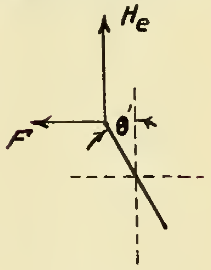


Fig. 13.

$$\mathcal{H}_e\, m'\, l \sin\theta' = F\, l\, m' \cos\theta' ,$$

and,
$$F = \mathcal{H}_e \frac{\sin\theta'}{\cos\theta'}$$

$$= \frac{\mathcal{J} s}{r_1^2}\left\{1 - \left(\frac{r_1}{r_2}\right)^3\right\} .$$

Therefore,
$$\mathcal{J} = \frac{\mathcal{H}_e\, r_1^2}{s\left\{1 - \left(\frac{r_1}{r_2}\right)^3\right\}} \tan\theta' .$$

The flux density is now readily determined from,

$$\beta = \mathcal{H} + 4\pi\mathcal{J} ,$$

and the value of field for any value of β from,

$$\mathcal{H} = \frac{4\pi n' I}{10}$$

where n' = no. of turns per cm. length of M .

and I = current thru M in amperes.

If the errors of observation and of measuring distances corectly are small, this method may be accepted to be as accurate as any providing the ratio of the length of the specimen to its cross-sectional area is large. The poles produced near the ends of the specimen oppose the inducing field, the value of the counter field set up depending on the size of the poles and the distance between them. If the ratio of length to breadth is greater than 100 , the correction for $\mathcal{H}$ is small but must be made for accurate work; for a value of 300 or more it is negligible; for a value of less than 50 it is great and cannot be made exactly. If $\mathcal{H}'$ = field computed from $\frac{4\pi n' I}{10}$, then the true inducing field is given by $\mathcal{H} = \mathcal{H}' - N\mathcal{I}$ where, N is the correction factor depending on the ratio of length to breadth. Carhart and Patterson give the following table from which a curve may be plotted.

Ratio of length to breadth.	N
50	0.01817
100	.00540
200	.00157
300	.00075
400	.00045
500	.00030

(b) Determination of Steinmetz' Coefficient (η).

The fundamental equation of the transformer is:

$$E = \sqrt{2} \quad \pi f n \Phi \; 10^{-8},$$

where,

E = effective value of the electromotive force.

f = frequency of alternations per second.

n = the number of turns.

Φ = maximum value of the total flux.

To get the above relation, one c.g.s. unit of e.m.f. is generated in each turn when the flux changes at the rate of one line per second, i.e.,

$$e = \frac{d\phi}{dt},$$ but if at any instant the flux is expressed by the relation,

$$\phi = \Phi \sin \omega t, \qquad (1).$$

where $\Phi = A \beta_{max}.$

A = net cross-sectional area of flux path or core.

β = maximum value of the flux density in lines per unit area.

$\omega = 2\pi f$; or angular velocity, having a value $2\pi f$.

By substitution, we have,

$$e = \frac{d\,(\Phi \sin \omega t)}{dt} = \Phi\,\omega \cos \omega t, \qquad (2).$$

e, is the maximum when $\cos \omega t = 1$; hence the maximum value of

e is :

$e = \Phi\ \omega$, and the effective or true value of e.m.f. per turn, expressed in c.g.s. units is:

$e_{eff.} = \Phi\ \omega \div \sqrt{2}$, but when n turns are used, the effective value of e.m.f. expressed in volts is:

$$E = \frac{\Phi\ \omega\ n}{\sqrt{2}\ \ 10^8} = \frac{\Phi\ 2\ \pi\ fn}{\sqrt{2}\ \ 10^8} = \frac{\sqrt{2}\ \ \pi\ f\ n\ \Phi}{10^8} = 4.44\ f\ n\ \Phi 10^{-8} \quad (3)$$

From the above $\beta_{max.} = \Phi \div A$.

Going back to the fundamentals, for a differential unit of time we have an expression for ergs, i.e.;

$$d\ W_{ergs} = e\ i\ d\ t = W, \text{in ergs per dt.} \quad (4)$$

e = c.g.s. or ab volt. i = ab ampere, and dt = dif. time.

In a transformer having iron in its circuit, we do not have a sine wave of current when a sine wave of e.m.f. is impressed; however, the relation

$$e = -\ n\ \frac{d\Phi}{dt} \quad (5)$$

holds for induced voltages, and if the flux wave is expressed by

$$\phi = \Phi\ \sin\ \omega\ t \quad (5')$$

then,

$$\frac{d\phi}{dt} = \omega\ \Phi\ \cos\ \omega\ t, \quad (6)$$

and from (5') it will be seen that in this case the e.m.f. wave will be of similar form but 90° behind in time.

But, $\Phi = A\ \beta_{max.}$

then,

$$e_{imp.} = n\ A\ \frac{d\beta_{max.}}{dt} \quad (7)$$

Also, $$\mathcal{H} = \frac{4\pi n i}{l} \quad \text{or,} \; i = \frac{\mathcal{H} l}{4\pi n} \qquad (8)$$

where,

$\mathcal{H}$ = magnetic field intensity and is expressed in gilberts per unit length.

$$\mu \mathcal{H} l = F = \beta l \qquad \mu \mathcal{H} = \beta .$$

l = length of magnetic path in e m.

$\frac{d\beta}{dt}$ = rate of change of flux in one unit section or sq. cm.

$\frac{d\phi}{dt}$ = rate of change of flux.

n = number of turns.

F = magnetizing force in gilberts.

Substituting (7) and (8) in (4), we get,

$$W, \text{ in ergs per dt.} = nA\frac{d\beta}{dt} \cdot \frac{\mathcal{H} l}{4\pi n} \cdot dt = \frac{Al}{4\pi} \mathcal{H} \, d\beta \; , \qquad (9)$$

or, the ergs per cycle are

$$W = \int \frac{Al}{4\pi} \mathcal{H} \, d\beta = \frac{Al}{4\pi} \int \mathcal{H} \, d\beta = \frac{\text{Vol.of Path}}{4\pi} \int \mathcal{H} \, d\beta . \qquad (10)$$

Now P_h in ergs = loss due to hysteresis per second,

or, $$P_h = \eta V f \beta^x . \qquad (11)$$

We can equate the two expressions by multiplying (10) by f, which gives loss in ergs per second, i.e.,

$$P_h = \eta V f \beta^x = \frac{Vf}{4\pi} \int \mathcal{H} \, d\beta ,$$

that is,

P_h = loss per cycle, per c.c.m. of the iron, in ergs = $\eta \beta^{1.6}$

Also, $$\eta = \frac{1}{4\pi \beta^x} \left(\int \mathcal{H} \, d\beta \right). \qquad (12)$$

The term in brackets is derived from the area of the hysteresis loop for the iron in question, and may be determined as follows:

$$\mathcal{H} = \frac{4\pi n i}{10\, l}, \text{where } n = \text{no. of turns} = 132$$

l = mean length of the magnetic circuit in cm. =61 cm

i = amperes, (10 in the above equation reducing from ab amperes to amperes).

$$\mathcal{H} = \frac{4\pi \cdot 13.2 \cdot .583}{10 \times 24'' \times 2.54} = \frac{2.2 \times .583 \times \pi}{2.54} = .505\,\pi \text{ Gausses}$$

$\mathcal{H}$ = 1.588 Gausses = Maximum $\mathcal{H}$ corresponding to $\beta_{max.}$

= Gausses per c.m. length of circuit.

From the above, $\beta_{max.} = \frac{\Phi}{A}$. Also, $\beta = \mu \mathcal{H}$.

From the fundamental equation of the transformer

$$\Phi = \frac{E\, 10^8}{4.44\, fn}.$$

By substitution of the proper values, we get,

$$\Phi = \frac{220 \times 10^8}{4.44 \times 60 \times 132} = \frac{220}{4.44 \times 6 \times 1.32} \times 100,000$$

Φ = 625 000 lines = flux at normal excitation.

$$\beta_{max.} = \text{induction per sq. cm.} = \frac{625\ 000}{A},$$

where A = the cross section of the path, and for the simplest case should be constant throughout the magnetic circuit.

The common application of the equation (12) for η is where we have the equal section as stated above and assume $x = 1.6$, which is fairly accurate for most work. From (12)

$$\eta = \frac{1}{4\pi\beta^{x}} \left(\int \mathcal{H} \, d\beta \right) = \frac{1}{4\pi\beta^{x}} \Big[\text{(area of the}$$

hysteresis loop in square centimeters for the value of Φ which will give the β to be used) X (fraction of β per centimeter length of scale to which the loop is plotted) X (the value of $\mathcal{H}$ per unit (cm.) of scale (usually abscissae) to which the loop is plotted)$\Big]$.

Example; If from the loop shown, $\beta_{max.}$ is 9,000 lines per square centimeter and is represented by an ordinate equal to three centimeters, and $\mathcal{H}$ per centimeter length of magnetic circuit in gilberts = 1.5 and is represented by an abscissa equal to two centimeters; then, if the area of the loop is four square centimeters,

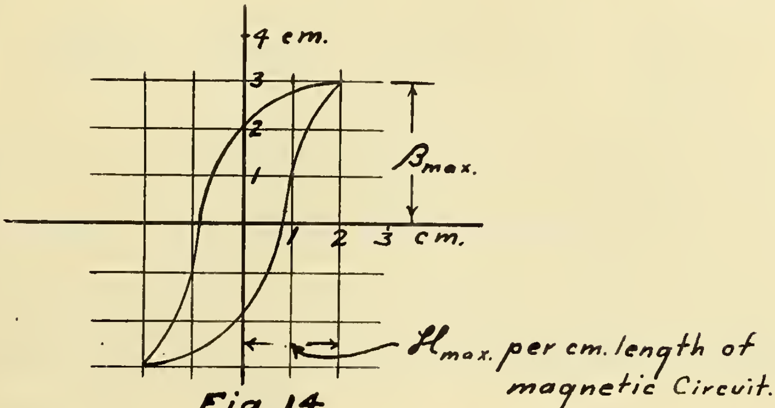


Fig. 14.

$$\int \mathcal{H} \, d\beta = \left(4 \times \frac{9000}{3} \times \frac{1.5}{2} \right).$$

The transformer considered in this treatise is by no means simple to handle in determining the hysteresis loss from a formula which gives η as does (12) above. To solve successfully the above equation for η with this transformer, it would be necessary to have a β and $\mathcal{H}$ curve for the iron from which μ could be found for any value of β , also hyster-

esis loops for the maximum densities that would exist in each section of the magnetic path of the transformer under normal existing current.

The reluctance of a path is directly proportional to its length and inversely proportional to the product of its cross-section and μ , i.e., Reluctance = $\frac{l}{\mu s}$; this is shown by the relation

$$\Phi = \frac{F}{\text{Rel.}} = \frac{\mathcal{H} l}{\text{Rel.}} = \mu \mathcal{H} s$$

or, cancelling,

$$\frac{\mathcal{H} l}{R} = \mu s \mathcal{H}. \quad \text{Hence, Rel.} = \frac{l}{\mu s}$$

Also, the total m.m.f., (F) may in the case of this circuit be considered as made up of three separate m.m.f.s, i.e., because of the different cross-sections and values of μ at the different densities we would have to express the total m.m.f. of the transformer thus:

$$F = F_1 + F_2 + F_3 = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \frac{l_3}{\mu_3 A_3}, \quad \text{where A}$$

takes the place of s.

By substituting the values of the terms in the second half of the equation, the total m.m.f. could be separated into three parts, F_1, F_2, and F_3, each of which would be the actual fraction of the total m.m.f. necessary to send Φ through that part of the circuit. If F_1, F_2, and F_3 are each divided by the lengths of their respective paths, the values of $\mathcal{H}$ in gilberts per centimeter length will be obtained, which values can be used in equation (12) to get μ for that section. The value of η for each section should be about the same in each

case for medium ranges of density. η being thus determined and β being known or obtainable, by using the relation $P_h = \eta\, V\, f\, \beta^x\, 10^{-7}$, it is possible to get the hysteresis loss in each part of the magnetic path.

At this point an approximate solution for η will be carried through for an average value of β and $\mathcal{H}$, using the general hysteresis loop as shown on curve sheet No. 4, represented by the smallest loop.

$$F = \frac{4\pi \cdot 132 \cdot .583}{10 \times 61} = 1.588 \text{ gilberts.}$$

F, per centimeter length of scale = $\frac{1.588}{2.54}$ = .625 gilberts.

Solving for an average β for the transformer,

A_1 = 8.64□" = 55.7 sq. cm. l_1 = 6.40625" = 16.28 cm.

A_2 = 12.93□" = 83.5 sq. cm. l_2 = 6.40625" = 16.28 cm.

A_3 = 16.17□" = 104.2 sq. cm. l_3 = 11.188" = 28.4 cm.

A better estimate of the length of the flux path through each of these sections is,

l_1 = 16.5. l_2 = 16.5. l_3 = 28.

55.7 x 16.5 = 920

83.5 x 16.5 =1376

104.5 x 28 = 2822

61 | 5218

85.6 = mean cross-sectional area of flux path. Then average $\beta = \frac{625\,000}{85.6}$ = 7,300 lines per square centimeter. β per centimeter length of scale = $\frac{7300}{2 \times 2.54}$ = 1,437

Area of loop = 2.0125 sq. in. = 12.98 sq. cm.

An average or approximate value of η, assuming that Steinmetz' exponent at this density and excitation is 1.64 which is very close as is shown by the curve from which x is found, is,

$$\eta = \frac{1}{4\pi \; 7{,}300^{1.64}} (12.98 \times .625 \times 1{,}437.) = \frac{1.167}{2720}$$

$$\eta = .000\,429$$

This value checks within four or five percent of the value found by the following method.

Where we have three different cross sections of iron as in this transformer, the total hysteresis loss can be expressed thus:

$$P_T = K' f \left(V_1 \beta^{x_1} + V_2 \beta^{x_2} + V_3 \beta^{x_3}\right).$$ Expressed separately,

$$P_{1h} = \eta V_1 f \beta_1^{x_1} 10^{-7},$$

$$P_{2h} = \eta V_2 f \beta_2^{x_2} 10^{-7},$$

$$P_{3h} = \eta V_3 f \beta_3^{x_3} 10^{-7}.$$

η will remain constant for all values of flux density around normal. Rewriting the equation for the total hysteresis loss and solvinv for η,

$$\eta = \frac{P_T}{\frac{1}{10{,}000{,}000} \times 60\left(V_1 \beta_1^{x_1} + V_2 \beta_2^{x_2} + V_3 \beta_3^{x_3}\right)}$$

V_1 = 907 cu. cm. β_1 = 11,200 lines per sq. cm.

V_2 = 1358 cu. cm. β_2 = 7,480 lines per sq. cm.

V_3 = 2600 cu. cm. β_3 = 6,000 lines per sq. cm.

P_T = 35.4 watts. = $K_h' \times .59$ = hysteresis loss at normal frequency and excitation.

P_e = 6.1 watts = 14.7% of total loss.

$x_1 = 1.7$; $x_2 = 1.65$; $x_3 = 1.6$.

$$\eta = \frac{35.4}{\frac{6}{100,000}\left[907 \times (11,200)^{1.7} + 1358 \times (7,480)^{1.65} + 2600 \times (6000)^{1.6}\right].}$$

η = .0004482.

Solving for the hysteresis losses in each part separately,

$P_{1h} = .0004482 \times 907 \times 60 \times (11,200)^{1.7} \times 10^{-7}$

P_{1h} = 18.65 watts.

$P_{2h} = .0004482 \times 1358 \times 60 \times (7,480)^{1.65} \times 10^{-7}$

= 9.03 watts.

$P_{3h} = .0004482 \times 2600 \times 60 \times (6000)^{1.6} \times 10^{-7}$

= 7.75 watts.

$P_T = P_{1h} + P_{2h} + P_{3h} = 18.65 + 903 + 7.75 = 35.43$

T_T = 25.4 watts.

Volume of core = 300 cu. in.

Weight of core = 84 lbs. approximately.

Hysteresis loss per pound of iron = $\frac{35.4}{84}$ = .422 watts

Eddy current losses = 17.2% of hysteresis.

Eddy current losses = 15% of the total iron losses.

(c) Separation of Losses Under Normal Operating Conditions

The data from the test at constant excitation, e.m.f. and frequency varying, was used to separate the hysteresis and eddy losses at normal frequency and excitation. (For reference, see Swenson and Frankenfield, Volume II). At constant excitation the equation

$$P_T = k_h f \beta^{1.6} + k_e f^2 \beta^2$$ may be

reduced to $P_T = k_h' f + k_e' f^2$; then by dividing through by f,

$$\frac{P_T}{f} = k_h' + k_e' f,$$ which is the equation of

a straight line. Plotting f as abscissae and P_T/f as ordinates, the straight line curve will cut the vertical axis at a distance K_h' above the origin. (See data, page 69, curve, page 89). To get P_T/f, the coreless was merely divided by the frequency as shown. By taking the ordinate at any point and multiplying it by the frequency at that point, the hysteresis loss is obtained. At normal frequency of 60 cycles the hysteresis loss was 35.4 watts, leaving the eddy loss = $\overset{P_T}{41.5} - \overset{P_h}{35.4} = \overset{P_e}{6.1}$ watts.

(1) Determination of Eddy Losses.

Eddy current loss is purely an electrical phenomenon. The currents are set up by a secondary induced e.m.f., and

hence are proportional to f and β. The loss then is proportional to f^2 and β^2. The extent to which these currents will exist is dependent entirely upon the shape and resistance of the iron. It has been found that the eddy losses in a laminated iron core vary directly as the square of the thickness of the laminations in centimeters. Then the loss due to eddy currents is,

$$P_e = \epsilon V f^2 l^2 \beta^2.$$

Given the relations,

$$E = 4{,}44\, f\, n\, \phi\, 10^{-8}, \text{ and } P_e = \epsilon V f^2 l^2 \beta^2,$$

where,

β is proportional to ϕ and is the maximum flux density,

l = thickness of the laminations,

f = frequency,

V = volume of iron in cu. cm.,

ϵ = coefficient, or constant, depending on the electrical conductivity of the iron.

n = number of turns,

E = e.m.f.

It is seen that E will vary with ϕ or β if f is constant; i.e., we can say, P_e varies with E^2; then, at constant frequency,

$$\frac{P_e \text{ at any } E}{P_e \text{ at normal } E} \qquad \frac{\text{any } (E)^2}{(\text{Normal } E)^2}$$

$$P_e \text{ for any value of } E = E^2 \left(\frac{6.1}{48{,}400}\right) = E^2(.000126).$$

Data for separation or determination of eddy losses is on page 68. Knowing the eddy loss in each case, the hysteresis loss was obtained by subtracting the eddy loss from

the core loss. The hysteresis loss as a percent of the total loss was found and plotted as ordinates against total loss as abscissae. It will be seen that this curve has a minimum point, and that with an increase of excitation or density beyond this point the hysteresis loss becomes a greater part of the total loss. In order to show that this curve will take such a shape as to have a point where the slope is zero, we will go to the hysteresis and eddy loss equations. The total loss,

$$P_T = P_h + P_e = k_1 \beta^x = k_2 \beta^2$$

Also,

$$P_h = k_1 \beta^x \text{ and } P_e = k_2 \beta^2$$

then,

$$\frac{P_h}{P_T} = \frac{k_1}{k_1 \beta^x + k_2 \beta^2}.$$

In solving for k_1 and k_2, the values of P_h and P_e at normal excitation are 35.4 and 6.1 watts respectively. From previous work β for average section = 7,300 lines per square centimeter.

$$P_h = 35.4 = k_1\ 7300^x. \quad \text{If } x = 1.6.$$

$$k_1 = \frac{35.4}{(7,300)^{1.6}} = 2.33 \times 10^{-5}.$$

$$P = 6.1 = k_2\ 7,300^2; \text{ then,}$$

$$k_2 = \frac{6.1}{7,300^2} = 1.15 \times 10^{-7}.$$

k_1 and k_2 depend upon η and V and hence can be considered constant for almost all conditions ordinarily considered in the

case of any one transformer. x, increases slightly with an increase of β as will be shown later, which causes a slight change or increase of P_h for increasing values of β over what would be given if $x = 1.6 =$ constant, as is ordinarily assumed. If x is taken as 1.6 and is assumed constant, and if values of β are substituted in the equations for P_h and P_e, and $\frac{P_h}{P_h + P_e}$ be plotted as ordinates against $P_h + P_e$, there will be no point on the curve where its slope is zero; that is, within finite limits. The following table will aid in checking this.

β	x	β^x	β^2	$K_1\beta^x$	$k_2\beta^2$	$K_1\beta^x + k_2\beta^2$	P_h/P_e
4000	1.6	579800	16000000	13.5	1.84	15.34	0.880
8000	"	1757600	64000000	41.0	7.35	48.35	.848
12000	"	3363500	144000000	78.5	16.56	95.06	.825
15000	"	4658000	225000000	108.5	25.87	134.37	.808
20000	"	7614000	400000000	177.5	46.0	223.5	.795
30000	"	14568000	900000000	339.5	103.5	443.0	.766
50000	"	32990000	2500000000	768.0	288.0	1056.0	.727

On the other hand we know that x does increase slightly with increase of β, so if we take values of x which are about as they would actually be, and again solve as above, the minimum point in the curve is accounted for and comes as would be expected. As a proof see the following table

β	x	β^x	β^2	$K_1\beta^x$	$k_2\beta^2$	$K_1\beta^x + k_2\beta^2$	P_h/P_e
3500	1.58	468300	12240000	10.91	1.41	12.32	0.885
5500	1.59	891000	30250000	20.76	3.48	24.24	.856
7500	1.60	1585000	56250000	36.92	6.47	43.39	.852
9000	1.61	2312000	81000000	53.9	9.32	63.22	.853
10500	1.62	3230000	110250000	75.3	12.69	88.00	.855

in which calculations have been carried out for the values of x as shown.

(d) Determination of Steinmetz' Exponent.

From the laws which govern hysteresis loss in iron,

$$P_h = \eta V 8 \beta^x 10^{-7} = P_{Total} - P_e$$

$$= k_h \beta^x = k'_h E^x.$$ If f, , and V are constant,

$$P_h = k'_h + x \log E.$$

x may be found from this relation. Since this is a straight line equation,

$$x = \frac{\log P_h - \log k'_h}{\log E} = \tan \phi,$$ where ϕ is the angle that the line makes with the axis of log E. Since $\log k'_h$ is a constant, if a curve be plotted with $\log P_h$ as ordinates and log E as abscissae, x will be the tangent of the angle which this line makes with the abscissa. These values have been plotted and the value of the exponent, x, found to be a variable quantity. For high values of density

(1) $x = 2.12$ for very high values of flux density.

(2) $x = 1.74$ for medium high values of flux density.

(3) $x = 1.68$ for normal values of flux density.

(4) $x = 1.57$ for low values of flux density.

The first three of these values are high though they are exactly as the curve represents. See curve sheet 9 , and data sheet 68 . It is more probable that the above values

should be approximately (1)'1.65; (2)' 1.62; (3)' 1.6; and (4)' 1.58 or possibly lower. However, this is nothing more than an estimate made from a knowledge of the customary limit within x usually remains. The high values taken from the curve are probably due to errors in the wattmeter readings which come from very low power factors. An error in the readings would effect the shape of the curve.

Effect of Hysteresis on Exciting Current.

When an alternating e.m.f. is impressed across a reactance coil, the current lags practically 90 degrees behind the e.m.f. If the coil has resistance as well as reactance, the angle of lag is less, depending on the ratio of resistance and reactance. The hysteresis loss in a transformer has in a manner the same effect as a resistance. Work is required to magnetize the iron and the angle of lag of current behind e.m.f. is decreased until, just as the point of $\beta_{max.}$ is approached, the current wave falls back, assuming nearly its proper phase relation to the e.m.f., that is, considering reactance alone as affecting this relation. Work is also required to demagnetize the iron and magnetize it to a maximum in the opposite direction. This, of course, requires an energy component of current in phase with the e.m.f. The resultant wave, therefore, advances from its 90 degree relation to about 30 or 40 degrees behind the e.m.f. The print of oscillogram No. 2, page 84, shows this relation.

Curve sheet No. 10, page 92, illustrates the method of finding the current wave when the hysteresis loop is drawn to the percent scale, using the maximum value of the exciting current as 100% and the value of $\beta_{max.}$ resulting as 100%, current being plotted as abscissae and flux density as ordinates. This hysteresis loop was taken from the general loop at normal excitation for the three sections of the transformer, given on curve sheet No. 4, page 87. A sine wave of e.m.f. is assumed to be impressed across the

transformer and this also is plotted on the percent basis. The sine wave of flux resulting is shown lagging 90 degrees behind the e.m.f. Since we have the hysteresis loop and the flux wave, the current wave may be plotted by construction. When the flux is a maximum, the current is a maximum, and the maximum points of current and flux waves will come together. When the flux has decreased to point A shown on the hysteresis loop and corresponding point A' shown on the flux wave, it is seen from the abscissa of the hysteresis loop that the value of current is 40% of maximum, and positive. It is, therefore, plotted at point A" directly under A'. This is the method of construction used to determine the wave of exciting current. The form of the current wave shows that hysteresis has a very considerable effect on the wave which would otherwise, neglecting eddy currents and low resistance of coils, be a sine wave. In this construction the eddy current loss, which is only about 15%, is neglected entirely. This loss also requires energy current and still farther decreases the angle of lag of current behind the e.m.f. wave. From the oscillogram it is seen that even at zero value of e.m.f. the current lags some 10 degrees behind. As a comparison, the current wave obtained from the oscillogram is plotted on the same sheet with the constructed current wave. The maximum value of this wave should be about 10 degrees behind that of the constructed wave but for the sake of comparison they are made to coincide.

Considering that the eddy current loss is not taken into account and that this is only the general hysteresis loop of the transformer, these waves compare very well.

Hysteresis a Function of Resistance in the Line.

The effect of resistance in the line on the hysteresis loss of a transformer may aptly be shown by analizing a general case. Assume that we have a sine wave of e.m.f. at the terminals of the generator, and that the exciting current of the transformer is of such a form as that of the constructed wave of current on curve sheet No. 10 , page 92 . This current flowing thru the resistance of the line will cause an IR drop in phase with the current. The condition is shown on curve sheet No. 11 , page 93 . The e.m.f. consumed by the line resistance will be of the same shape as the current wave. The wave actually impressed across the transformer is the difference between the sine wave and this IR drop wave. It is readily seen that it is more flat than a sine wave of equal effective value, and hence has a greater area. By calculation, the form factor was found to be 1.107 which is slightly less than that of a sine wave, the form factor of a sine wave being 1.112 .

Since the hysteresis loss in a transformer varies inversely as the form factor to the 1.6 power, it is determined that the effect of resistance in the line is to increase the hysteresis loss in the transformer.

Discussion of Proper Flux Density.

(a) Effect on Constant Losses.

It is desirable to design any machine so as to obtain the maximum efficiency at the load under which it is most usually operated. Of the two losses of the transformer, copper loss and core loss, the first depends upon the amount of copper and the second on the amount and kind of iron, thickness of laminations, etc. For maximum efficiency the copper, or variable loss, equals the constant core loss, hence the weight of copper and iron increases in a constant ratio as the efficiency increases.

Curves as shown in Fig. 15 might be plotted with percent combined weight of copper and iron as abscissae, taking as 100% the value that gives practically 100% efficiency; making the ordinates for No.1 the cost in percent of cost at practically 100% efficiency, and the ordinates for No.2 the percent efficiency of the transformer. The cost will, of course, increase directly as the weight of copper and iron, neglecting the decrease of cost of construction with increase of size up to a certain limit, while the efficiency will increase at a much faster rate until the point of tangency of a line parallel to No.1 is reached. The transformer will

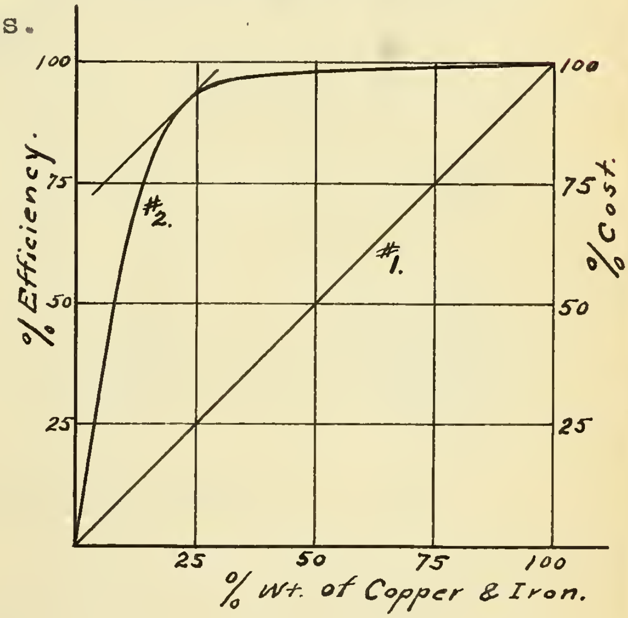


Fig. 15.

naturally be built to give maximum efficiency somewhere around this point of tangency, the cost of power coming in as another factor.

That the maximum efficiency for any given transformer is at that percent of load where the constant losses equal the variable losses, may be very simply proven by taking the first derivative of the efficiency equation and setting it equal to zero.

$$y = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$= \frac{p\,e\,i}{p\,e\,i + C + i^2r} \quad ,$$

where,

y = efficiency,
p = power factor of load,
e = secondary voltage,
i = " current,
r = resistance of secondary plus resistance of primary in terms of secondary,
C = core loss in watts, which is constant.

Taking the first derivative, with respect to the current as the variable, and setting it equal to zero gives:

$$\frac{dy}{di} = \frac{(pei + C + i^2r)\,\dfrac{d(pei)}{di} - (pei)\,\dfrac{d(pei + C + i^2r)}{di}}{(\,pei + C + i^2r\,)^2} = 0$$

$$p^2e^2i + peC + pei^2r - p^2e^2i - 0 - 2pei^2r = 0$$

$$peC - pei^2r = 0$$

$$C = i^2r,$$

or, core loss = copper loss.

The copper loss of this transformer equals the core loss at about 65% of full load. From the efficiency curve this is seen to be the point of maximum efficiency.

The core loss depends upon two factors, hysteresis loss and eddy-current loss, the first of which increases as $\beta_{max.}^{1.6}$ and the second as $\beta_{max.}^{2}$. Since the hysteresis loss is usually between 80 and 85 percent of the total core loss, this total loss will increase as $\beta_{max.}^{z}$, where z depends upon the value of $\beta_{max.}$. It increases as $\beta_{max.}$ increases. For the flux density usually employed z lies somewhere between 1.8 and 1.9 ; therefore, it is evident that because of its important relation to the constant losses, care should be exercised in choosing the proper flux density.

In making this choice of the proper flux density, as regards losses, the load factor is considered. If this is large, then a density that will give a maximum efficiency at nearly full load will be used; if small, a lower density will be used in order to decrease the core loss and improve the all-day efficiency. This was undoubtedly the main point considered in using such a low density in the transformer tested, because a density was used lower than that necessary to keep the charging current down within a reasonable value.

(b) Effect on Charging Current.

No mechanical device for opening or closing an alternating current circuit at the zero of the current wave has yet been devised, and it is very improbable that any such will be because of the very rapid change of the current wave even in a 25 cycle system. To open or close such a circuit within 10 degrees of the zero value of the current wave would require that the device be adjusted to operate within $\frac{1}{25 \times 36} = \frac{1}{900}$ th of a second, which is inconceivable for even the most refined mechanical contrivance.

The natural tendency of a circuit, however, is to open at the zero value of the current wave, for the arc formed at the terminals will usually hold until the current is very near zero. This is because the switch points have an oppurtunity to travel but a very short distance in space while the current wave changes at most but one-half of a cycle. The most likely point of the wave other than zero at which the current would break, would be at a low value of the increasing positive or negative part of the wave, as points C or C', Fig. 16. This is not the case on closing the circuit. Then no arc is formed and the electrical circuit is completed instantaneously when the switch points come in contact.

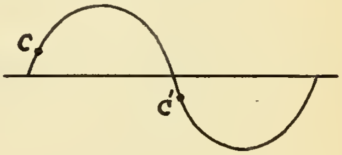


Fig. 16.

In transformer operation the ideal condition would be always to open and close the switch at the zero value of the current and at the same point of the wave, as at A on the current wave and the corresponding point, A', on the hysteresis

loop of the transformer, figures 17 and 18. Then the current

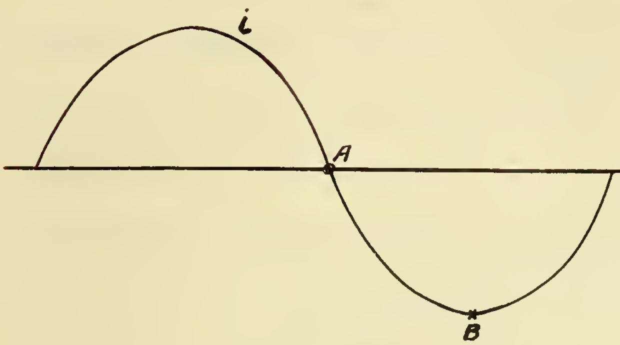


Fig. 17.

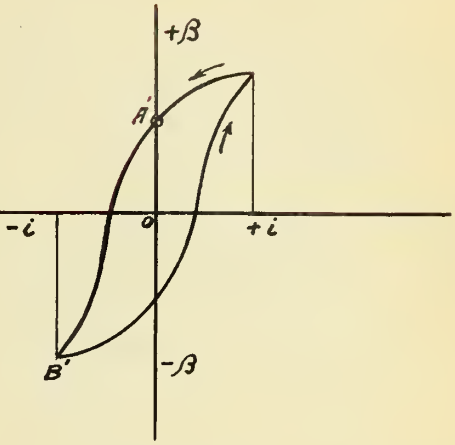


Fig. 18.

would start from where it left off and the hysteresis loop continue to repeat itself as if undisturbed except for the effects which inductance and capacity of the transformer have on the charging current.

This ideal condition is not obtainable because of mechanical considerations, as explained above; therefore, the transformer must be designed so that damage will not be done when the circuit is closed under the worst possible conditions. One of these conditions occurs when the switch is closed at the point, B, of the current wave and corresponding point, B', on the hysteresis loop, providing the circuit was opened at point A . The instantaneous value of flux required to give the proper counter e.m.f. will be about 170% of the normal since the retentivity with a closed iron circuit is usually such that the point, A' , is at approximately 70% of $\beta_{max.}$.

If it is assumed that $\beta_{max.}$ = 100,000 lines per sq.in., and β at saturation = 130,000 , then such a value of exciting current will be required as will produce a M.M.F.

sufficient to saturate the iron and send 40,000 lines per sq.in. thru space. By continuing the saturation curve for Apollo Electric Special Steel, (curve sheet No. 12 , page 94 ,), it is found that approximately 800 A.T. per inch of length are required at saturation, if saturation is assumed at 130,000. The mean length of flux path in the transformer tested was 24 inches. For this length (800 × 24) or 19,200 A.T. will be required to saturate the iron.

$$\text{M.M.F.} = \phi \mathcal{R}$$

$$= A' \beta' \frac{l'}{\mu A'}$$

$$= .394 \frac{A \beta l}{A}$$

$$= 0.394 \beta l$$

where, $\mathcal{R}$ = reluctance of path,
l' = length in centimeters,
A' = area " sq. " ,
β' = flux per " " ,
μ = permeability, (for air = 1).
l = length in inches,
A = area in sq. " ,
β = flux per " " .

Therefore,

$$\frac{4\pi AT}{10} = 0.394 \times 40,000 \times 24.$$

$$\text{A.T.} = \frac{0.394 \times 40,000 \times 24 \times 10}{4\pi}$$

$$= 300,700.$$

Ampere-turns required above saturation	=	300,700.
" " " for "	=	19,200.
Total ampere-turns required	=	319,900.

Full-load secondary current for the transformer tested = 22.7 amperes, and secondary turns = 132. Therefore, with this

kind of steel and the densities specified, and exciting current equal to $\frac{319{,}900}{22.7 \times 132} = 106.7$ times normal full-load current would flow theoretically. Of course, the transformer resistance and reactance would cut this down to probably not more than 10 or 12 times full-load.

This example brings out clearly the importance of using the proper flux density in order to limit the value of charging current under the worst conditions of closing the circuit.

If a value of $\beta_{max.} = 76{,}500$ had been used, then under the worst conditions the iron would just reach saturation and the theoretical value of the charging current would equal,

$$\frac{19{,}200}{22.7 \times 132} = 6.41 \text{ times normal full-load}$$

current. This current continues for so short a time that the I^2R loss will not be dangerous in causing excessive heating, and consequently the likelihood of burning out a coil will be small.

It is now interesting to note the other extreme, of which the transformer tested was an example. Here the value of.

$\beta_{max.} = 72{,}300$ in the center leg,

$= 38{,}700$ " " ends,

$= 48{,}300$ " " outside legs.

Under the worst conditions of closing the circuit,

$\beta_{max.}$ in the center leg $= 72{,}300 \times 1.7 = 122{,}800$ lines/sq.in.

$\beta_{max.}$ " " ends $= 38{,}700 \times 1.7 = 65{,}750$ " / " ".

$\beta_{max.}$ " " outside legs $= 48{,}300 \times 1.7 = 82{,}100$ " / " ".

Since no saturation curve was obtained for a uniform cross-section, the curve for Apollo steel will be used in this case also, altho it is apparent from the general saturation curve, the one obtained where the cross-section differed, that it requires even a less number of ampere-turns for this special silicon steel than it does for the Apollo steel.

From the curve, A.T. required per in. length at $\beta_{max.}$ =122800.=550.
" " " " " " " " " " $\beta_{max.}$ = 65750.= 10.
" " " " " " " " " " $\beta_{max.}$ = 82100.= 24.

Mean length of flux path in center leg = 6.5 inches.
" " " " " " ends = 11.0 " .
" " " " " " outside legs = 6.5 " .

A.T. required for center leg = 6.5 × 550 = 3575.
" " " " ends = 11.0 × 10 = 110.
" " " " outside legs = 6.5 × 24 = 156.
Total A.T. required ------------------= 3841.

From the above, the maximum current under the worst conditions

$$= \frac{3841.}{22.7 \times 132} = 1.282 \text{ times normal full-load current.}$$

In this case it is the center leg which requires practically all of the magnetomotive force.

Besides the danger of a high value of the charging current in causing burn-outs in the transformer itself, there must be considered its effects in the line, and instruments, such as ammeters and wattmeters, in series with the line. It will also cause trouble with the oil-switch operation. The oil-switch relays, in connection with current transformers, are

regulated to trip at a dangerous overload. This heavy charging current will cause them to trip as quickly as the oil-switch is thrown in until the switch happens to close at a favorable part of the wave. There are thus seen to be a number of conditions limiting the value of the charging current, and hence the choice of the proper density.

In large transformers especially, it is usual to have a reactance of about 5% . This, together with the resistance, sets a limiting value on the current that can flow with a given value of e.m.f. impressed. In lighting transformers, such as the one tested, the reactance is kept down to a low value, in order to make the regulation good. Another consideration, that of core loss, leads to keeping the flux density low, so there is no danger of a large charging current, and hence no need of a high reactance. It is in large transformers, worked at a relatively high density so as to save iron, that the consideration of the proper reactance as a means of limiting the value of the charging current comes in.

Calculation of Transformer Regulation.

The percent regulation of a transformer is defined as the change in secondary voltage from full-load to no-load, expressed as a percent of the full-load secondary voltage taken as 100 .

The calculation is essentially the same as for the regulation of a transmission line without capacity, the energy current for core loss, and magnetizing current making it but slightly different.

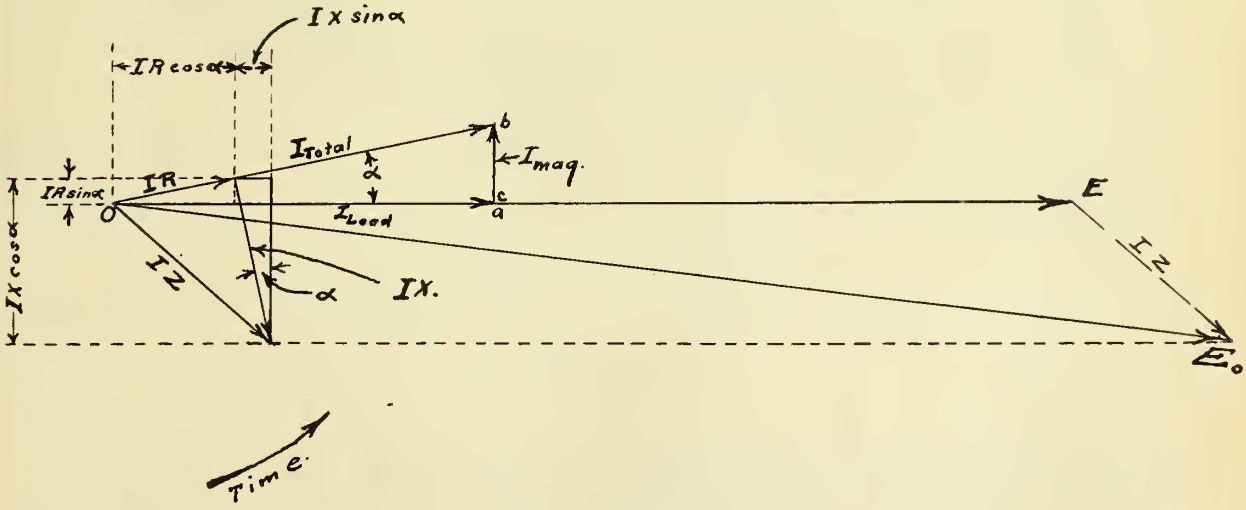

I x sin α
IR cos α
b
I_Total
α
I_mag.
E
IR sin α
IR
O
c
a
I_Load
IZ
α
IX.
I x cos α
IZ
E_o
Time.

Fig. 19.

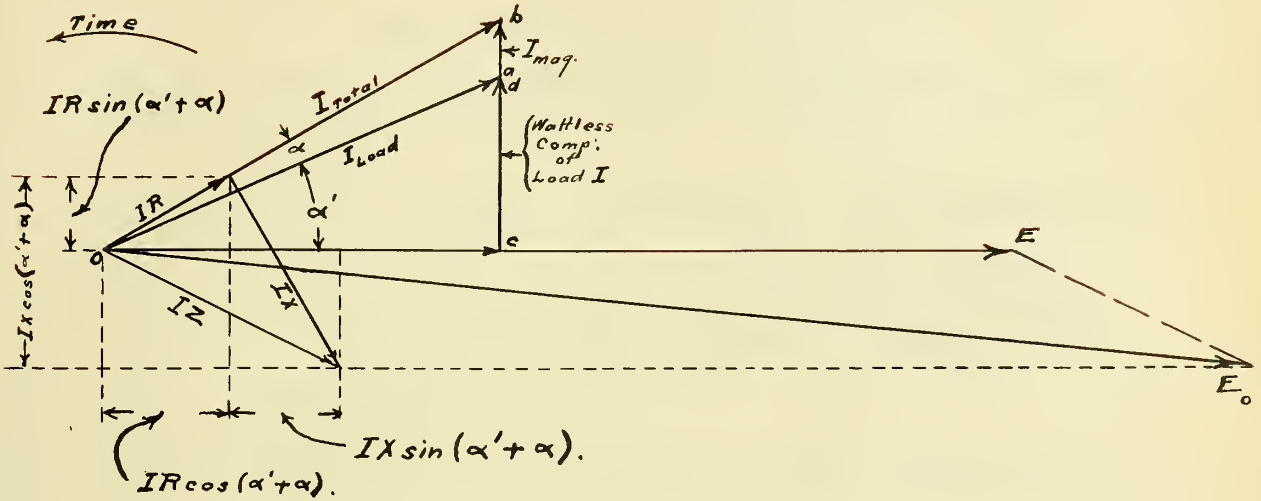

Time
b
$I_{mag.}$
a
d
I_{Total}
$IR\sin(\alpha'+\alpha)$
α
I_{Load}
Wattless Comp. of Load I
IR
α'
o
c
E
$IX\cos(\alpha'+\alpha)$
IX
IZ
E_o
$IX\sin(\alpha'+\alpha)$.
$IR\cos(\alpha'+\alpha)$.

Fig. 20.

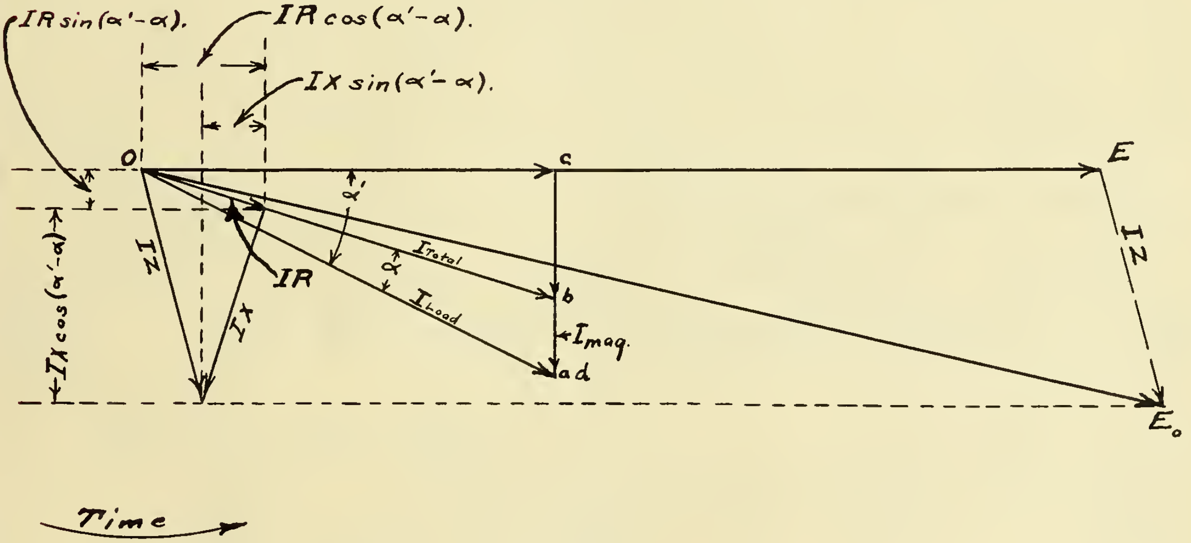

$IR\sin(\alpha'-\alpha)$.
$IR\cos(\alpha'-\alpha)$.
$IX\sin(\alpha'-\alpha)$.
O
c
E
α'
I_{Total}
α
IZ
IR
$IX\cos(\alpha'-\alpha)$
IX
b
I_{Load}
$I_{mag.}$
a d
IZ
E_o
Time

Fig. 21.

Figures 19, 20, & 21 show respectively the electromotive force diagrams for non-inductive, inductive, and capacity loads on the transformer. All values are reduced to percent with full-load current, and full-load voltage at the secondary, as 100. Secondary current, e.m.f., resistance, and reactance are expressed in equivalent primary values.

OE = secondary full-load voltage in terms of primary = 100%.

Oa = load current in the secondary in terms of " = 100%.

ab = wattless component of exciting current = magnetizing current.

Oc = energy component of secondary load current plus a small energy component of exciting current.

cd = wattless component of secondary load current.

Ob = total current.

IR = total % resistance drop in the transformer.

IX = " % reactance " " " " .

IZ = " % impedance " " " " .

E_o = E + IZ = voltage impressed on primary = secondary no-load voltage in terms of primary. (practically)

From Fig. 19 it is seen by means of trigometric relations that,

$$E_o = \sqrt{(E + IR\cos\alpha + IX\sin\alpha)^2 + (IR\sin\alpha - IX\cos\alpha)^2}.$$

Similarly, from Fig. 20 ,

$$E_o = \sqrt{\left\{E + IR\cos(\alpha'+\alpha) + IX\sin(\alpha'+\alpha)\right\}^2 + \left\{IR\sin(\alpha'+\alpha) - IX\cos(\alpha'+\alpha)\right\}^2}.$$

And from Fig. 21 ,

$$E_o = \sqrt{\left\{E + IR\cos(\alpha'-\alpha) - IX\sin(\alpha'-\alpha)\right\}^2 + \left\{-IR\sin(\alpha'-\alpha) - IX\cos(\alpha'-\alpha)\right\}^2}.$$

$\left.\begin{array}{l}\cos\alpha\\ \cos(\alpha'+\alpha)\\ \cos(\alpha'-\alpha)\end{array}\right) =$ (power factor at the transformer. (They may be considered as the power factor of the load since the magnetizing current is such a small % of the full-load current.)

Without appreciable error $(\cos\alpha)$ may be considered equal to 1 , $\cos(\alpha' + \alpha) = \cos\alpha'$, and $\cos(\alpha' - \alpha) = \cos\alpha'$.

$\sin\alpha$ = ratio of % magnetizing current to % full-load current, called the inductance factor of the transformer.

$\sin\alpha'$ = ratio of wattless component of load current to total load current, called the inductance factor of the load.

$\sin(\alpha' \pm \alpha)$ = ratio of total wattless current to total current taken by transformer, the inductance factor of transformer and load.

Denote power factor of load by $p = \cos\alpha'$.
" inductance " " transformer by $q = \sin\alpha$.
" " " " load " $q' = \sin\alpha'$.

Since $\sin\alpha$ is very small, α is very small, and an inappreciable error will be introduced by considering $\sin(\alpha' \pm \alpha) = \sin\alpha' \pm \sin\alpha$. Introducing these values in the above equations gives:

For non-inductive load,

$$E_o = \sqrt{(E + IR + qIX)^2 + (qIR - IX)^2} ,$$

For inductive load,

$$E_o = \sqrt{\{E + pIR + (q' + q)IX\}^2 + \{(q' + q)IR - pIX\}^2} ,$$

For capacity load,

$$E_o = \sqrt{\{E + pIR - (q' - q)IX\}^2 + \{-(q' - q)IR - pIX\}^2} .$$

IR is the total resistance drop in the primary and secondary expressed in percent of normal voltage, determined from full load currents in primary and secondary, and measured resistances.

IX is the total percent reactance drop of the transformer as determined from the copper loss test at normal current, where $IX = \sqrt{(IZ)^2 - (IR)^2}$, IZ being the voltage necessary to impress to force full-load current thru the impedance of the transformer, IR being the wattmeter reading divided by the current, or the resistance drop. This method gives the total reactance drop in volts.

q is determined from the core loss test, the magnetizing current being determined from $I_{mag.} = \sqrt{(I_{ex})^2 - (I_{he})^2}$, where I_{he} is the energy component of the exciting current, I_{ex}, taken to supply the hysteresis and eddy current losses at normal voltage and frequency.

$$I_{he} = \frac{\text{watts core loss}}{\text{impressed voltage}} .$$

The regulation of this transformer will be calculated for non-inductive loads only.

When the equations on page 54 are investigated it is seen that for a load P.F. of unity the quanties (qIR) and (qIX) are small. (qIR), being in the part of the equation which when squared will be small as compared with the first term, is negligible and may be disregarded in its effect on the value of E_o. In the first term (qIX) will be very small as compared with IR; hence, it is seen that, since at best the second term will have but a small effect on E_o, with a

load P.F. of unity the regulation depends almost entirely on the resistance of the transformer and is little influenced by the reactance.

For decreasing values of load P.F. the quantity (pIR) in the first term decreases, while $(q' \pm q)IX$ increases; therefore, the reactance comes more and more into effect. Usually the reactance is considerably larger than the resistance and so this change is more pronounced. For low power factors the reactance, and not the resistance, is the governing factor of the regulation.

Calculation of Regulation of Transformer at 50°C.

Resistance of primary.(2 coils in series). = 8.851 ohms.
Full-load pri. current.(2 " " "). = 2.27 amperes.
IR drop in primary. = 20.1 volts.
Normal voltage across pri.(2 coils in series). = 2200. "
Percent IR drop in primary. = 0.914 %

Resistance of secondary.(2 coils in series). = 0.1089 ohms.
Full-load sec. current.(2 " " "). = 22.7 amp.
IR drop in secondary. = 2.47 volts.
Normal voltage across sec.(2 " " "). = 220.0 "
Percent IR drop in secondary. = 1.124 %

Total percent IR drop. = 2.038 %

From Amp. vs. Volts curve on short-circuit: Impedance drop, IZ = = 56.5 volts.

To calculate % IX :

$(IZ)^2$ = 3190.

From Amp. vs. Watts curve on

short-circuit, copper loss, I^2R = 94. watts.

Resistance drop, $IR = \frac{\text{watts}}{I}$ = 41.4 volts.

$(IR)^2$ = 1713.

$(IX)^2$ = 1477.

(IX) = 38.45 volts.

Total % reactance drop = 1.748 %

Core loss from Watt vs. Volts

curve on open circuit. = 41.5 watts.

Exciting current from Saturation

curve, I_{ex} = 0.411 amperes.

Core loss energy current, I_{he}

$= \frac{41.5}{220}$ = 0.1887 " .

$I_{mag.} = \sqrt{(I_{ex})^2 - (I_{he})^2}$ = 0.365 " .

Inductance factor, $q = \frac{0.365}{22.7}$ = 0.01608

$\% E_o = \sqrt{(E+IR+qIX)^2 + (qIR - IX)^2}$ = 102.05 % .

(In the above equation for E_o the error introduced by neglecting the term (qIR) in the second member is insignificant).

% Regulation = 2.05 % .

Calculation of Full-Load and All-Day Efficiencies.

The efficiency of any electrical machine is defined as the ratio of the Output to the Input. The output of a transformer in watts is equal to the secondary voltage times the secondary current with a load power factor of unity. For any other power factor it is this product times the P.F.

The input is equal to the output plus the transformer losses, consisting of I^2R losses in the primary and secondary coils, and iron losses due to hysteresis and eddy currents in the core. These latter are known as the "core loss" , while the former are designated by the name, "copper loss". The core loss is constant for any given condition, and the copper loss varies as the square of the current.

The efficiency will be calculated for a power factor of unity.

Calculation of Efficiency of Transformer at 50°C.

Resistance of primary.(2 coils in series). = 8.851 ohms.
Full-load pri.current.(2 " " "). = 2.27 amperes.
I^2R loss in primary. = 45.6 watts.
Resistance of sec. (2 coils in series). = 0.1089 ohms.
Full-load sec.current.(2 " " "). = 22.7 amperes.
I^2R loss in secondary. = 56.0 watts.
Total copper loss. = 101.6 "
Core loss from open circuit test. = 41.5 "
Total losses. = 143.1 "
Input = Output + total losses. = 5143.1 "
Percent efficiency at full load. = 97.3 %

In calculating the all-day efficiency, it is usually assumed that the transformer is fully loaded for three hours of the day and remains idle for the rest of the twenty-four.

Three-hour copper loss. = 304.8 watts.
Twenty-four-hour core loss. = 966.0 "
Total losses for 24 hours. = 1270.8 "
Total output for 24 hours on the 3 hour basis. = 15000. "
Total input for 24 hours on the 3 hour basis. = 16270.8 "

All-Day Efficiency. = 92.2 % .

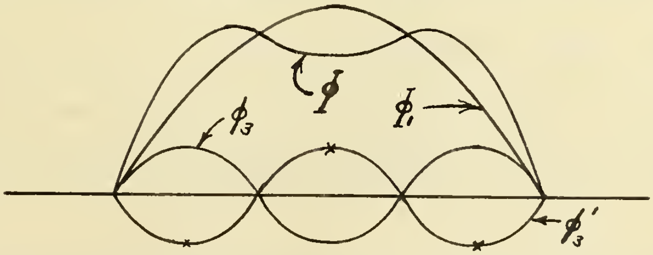
Three Phase Operation.

Since single phase transformers are used widely in three phase power work, it is important to make a study of them under these conditions. The three typical methods of connections used are shown under "Tests".

From previous work it has been found that where iron exists in an alternating current circuit, there must be either triple frequency voltage or triple frequency currents. Considering only the case where the e.m.f.s of a three phase circuit are practically 120 electrical degrees apart, it is evident that the fundamentals are also approximately 120 degrees from each other, while the triples or their multiples are 360 electrical degrees apart. This being true, they are, at any instant, in the same time phase relation and may be considered as leaving the alternator at the same time. On this assumption, which is practically the actual condition existing, it is evident that the triples will meet at the same instant as they enter the transformer and buck, or neutralize, each other, thus being eliminated. Actually, in the case explained above, the triples do not exist in the e.m.f. between lines. It is also true that triples or their multiples cannot exist in the current in the lines. This holds for the same reasons, essentially, as apply to the e.m.f. wave; that is, the triples neutralize each other and hence do not exist.

In the tests made on these transformers, the e.m.f. between lines was practically a sine wave, hence a sine wave

did not exist between line and neutral, and the current wave was essentially a sine wave due to the elimination of the triple. Since usually the triple is the most pronounced and the higher frequencies do not usually play so important a part, only the triples and their multiples will be considered. A sine wave of exciting current implies a peaked e.m.f. wave over the coil; i.e., between line and neutral, as a result of the effect of the iron. Because of the triple, the e.m.f. between line and neutral will have an effective value which is greater than the effective value of the e.m.f. between lines divided by the 3 , provided there is no circulating triple frequency current flowing in the secondary as would be the case if the secondary were a closed delta. If the secondaries are star connected, the same ratios of voltages will exist per coil and per line as would exist if these triples were not present. Now if the secondary is a closed delta, the voltage ratios change. A peaked e.m.f. wave corresponds to a flat topped flux wave and a sine wave of exciting current. The flat topper flux wave has a triple as shown in the figure, and the current wave is a sine curve. Φ_3 is the triple of the flat topped flux wave. With the secondary



ϕ_1 is the fundamental of the original flux wave, ϕ. ϕ_3' is due to the circulating triple frequency current in the secondary.

Fig. 22.

a closed delta, a triple frequency current, due to the triple voltage induced in the secondary, will flow; and this triple current will produce a flux nearly in phase, or at 180 degrees with the triple component of the original flat toppe~~r~~d flux wave (See Figure 22, $\Phi_{\frac{1}{3}}$). These two triples oppose and neutralize, and hence if the third, seventh, eleventh, etc., are neglected, will leave the flux wave, theoretically, very nearly a sine wave as represented by the fundamental shown above. This fundamental has a maximum value which is greater than the maximum value of the flat toppe~~r~~d flux wave. Since the hysteresis loss is $P_h = \eta V f \beta^x 10^{-7}$ and for a given frequency is dependent upon the maximum β^x, it is evident that the losses will be greater with the circulating current, whether this take place in a delta connected primary or secondary. (See data on coreless test, page 75, and 76). When the delta secondary is closed and the flux wave takes approximately a sine wave shape, the e.m.f.s per coil are also changed to a sine form, and the ratio of the line voltage to coil voltage is changed and is then $\sqrt{3} : 1$, which is the ratio most commonly assumed. The data will verify this, for example, with the open delta-secondary the line voltage was 174 while the corresponding coil voltage was 110. By closing the delta the line voltage changed to 190.7 for a coil voltage of 110.

Across the open delta a value of electromotive force will exist which is equal to one fundamental and twice the triple. (See data sheet No.75, Columns (a), and (b)). If

one corner of the delta be opened and the voltage measured, it will be three times the triple of one coil since the three coils are so connected as to cause the voltages of each coil to add up. These relations provide a means of measuring the triple harmonic in a three phase circuit. The methods of making the determinations of the value of the triple, fundamental, and the voltage across the delta are as follows, taking readings at normal voltage for the transformer.

Triple frequency voltage per coil of secondary = 1/3 of 130 volts = 43.3 volts = E_3.

Effective value of fundamental = $E_1 = \sqrt{E^2 - E_3^2}$

where

E = effective value of the voltage on one side of delta,

E_3 = effective value of the triple,

$$E_1 = \sqrt{109^2 = 43.3^2} = 100.3 \text{ volts.}$$

Voltage across the open delta $= E^1 = \sqrt{E_1^2 + (2E_3)^2}$

$$= \sqrt{10030 + (2 \times 43.3)^2}$$

$$= 132.5 \text{ volts.}$$

(For a discussion of the above method see Swenson and Frankenfield, Volume II, page 245.)

Assuming that there were no triple e.m.f. induced in the coils of the secondary, the voltage across the open delta or V would equal that across one of the coils, which would be the fundamental in this case; but where we have the triple the voltage across the open delta, it should equal the square root of the sum of the squares of one fundamental and two times the triple as given by the above formula.

An interesting thing observed while measuring the triple e.m.f. was that considerable variation in values could be gotten for any excitation by using voltmeters of different types. This is explained by the fact that any triple current that flows tends to produce a flux which opposes the triple flux change that generates the triple e.m.f. Now the resistances of voltmeters vary and also each takes some current which, though it is a small amount, is probably large compared with the triple current that would flow on short circuit of the delta. This variation in the current which will flow determines the amount of triple voltage that will be induced. Hence, we may conclude that the higher the resistance of the voltmeter used, the greater the triple will appear to be. This was proved in the tests made.

COPPER LOSS DATA.

Reading No.	Volts impressed across 2 high voltage coils in series. E.	Current in high voltage coils. I	Watts Copper Loss. W
1	10.0	0.38	2.6
2	15.0	.575	6.1
3	20.2	.775	11.3
4	25.2	.965	18.0
5	29.8	1.19	25.5
6	36.0	1.42	37.1
7	42.4	1.68	54.1
8	48.6	1.93	66.8
9	53.0	2.13	86.0
10	60.0	2.40	107.7
11	65.5	2.53	121.0
12	70.5	2.82	144.0
13	75.0	3.03	162.0
14	80.0	3.23	185.0
15	85.0	3.44	208.5
16	90.0	3.63	235.0
17	95.0	3.83	265.5
18	100.5	4.03	293.0
19	111.4	4.42	361.0
20	118.0	4.69	407.0

CORE LOSS DATA.[1]

Reading No.	Voltage across transformer. E	Exciting current. I	Core loss + copper loss	Copper loss at 25 °C. (R = 0.09945)	Core loss corrected for copper loss. W
1	100.0	0.15	11.2	Too small to correct for	11.2
2	125.0	.19	15.5		15.5
3	147.0	.225	20.3		20.3
4	161.5	.252	24.2		24.2
5	177.5	.28	29.0		29.0
6	193.0	.316	33.0		33.0
7	205.8	.357	36.9		36.9
8	214.0	.386	39.5		39.5
9	220.0	.411	41.5		41.5
10	238.0	.523	47.7		47.7
11	249.5	.614	52.0		52.0
12	267.0	.805	59.5	0.1	59.4
13	286.2	1.42	69.8	.2	69.6
14	304.5	2.48	78.9	.6	78.3
15	315.0	3.72	86.5	1.4	85.1
16	323.4	4.78	92.5	2.3	90.2
17	337.8	6.40			
18	349.5	8.6			
19	361.5	11.25			
20	368.4	12.6			
21	383.4	18.15			
22	393.0	21.75			
23	399.0	24.3			

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[1] Frequency constant at 60 ∼ .

CORE LOSS DATA.[1]

Reading No.	Voltage across transformer. E	Exciting current. I	Core loss. W
1	80.	0.25	10.3
2	100	.345	14.8
3	120	.545	21.3
4	130	.765	25.7
5	135	.965	28.1
6	140	1.13	29.4
7	145	1.56	32.2
8	148	1.95	33.9
9	155	3.04	37.3
10	160	3.87	40.2
11	166	4.70	43.6
12	180	8.60	
13	190	11.08	
14	195	13.6	
15	200	15.1	
16	206	16.7	

[1] Frequency constant at 30 ~ .

DATA FOR SEPARATION OF LOSSES.[1]

Reading No.	Volts imp. on trans. E	Iron loss = $P_h+P_e=P_T'$ (Values from core loss curve.)	$(E)^2$	$P_e = .000126\ E^2$ watts	$P_h = P_T' - P_e$ watts	% P_h of iron loss	Log E	Log P_h
1	100.0	11.2	10000	1.26	9.94	88.7	2.0000	0.9974
2	114.5	13.7	13110	1.65	12.05	88.0	2.0588	1.0810
3	125.0	15.7	15625	1.97	13.73	87.5	2.0969	1.1377
4	134.8	17.8	18171	2.29	15.51	87.2	2.1297	1.1906
5	147.0	20.7	21610	2.72	17.98	86.9	2.1673	1.2548
6	161.5	24.2	26080	3.29	20.91	86.5	2.2082	1.3204
7	177.5	28.7	31420	3.96	24.74	86.2	2.2492	1.3934
8	193.0	33.0	37250	4.69	28.31	85.8	2.2855	1.4519
9	205.8	36.9	42280	5.33	31.57	85.5	2.3134	1.4993
10	214.0	39.7	45750	5.76	33.94	85.5	2.3304	1.5307
11	220.0	41.5	48400	6.10	35.40	85.3	2.3424	1.5490
12	228.0	44.4	52000	6.55	37.85	85.3	2.3579	1.5781
13	238.0	48.4	56620	7.13	41.27	85.3	2.3766	1.6156
14	249.5	52.7	62200	7.83	44.87	85.2	2.3971	1.6520
15	267.0	60.1	71200	8.97	51.13	85.1	2.4265	1.7087
16	286.2	69.6	82000	10.33	59.27	85.1	2.4567	1.7728
17	294.0	73.7	86400	10.88	62.82	85.2	2.4683	1.7981
18	304.5	78.7	92700	11.68	67.02	85.2	2.4836	1.8262
19	315.0	85.1	99100	12.49	72.61	85.3	2.4983	1.8610
20	323.4	90.2	104500	13.17	77.03	85.4	2.5097	1.8867

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[1] Calculations made from single phase core loss data at constant frequency. (f = 60 ~)

DATA FOR SEPARATION OF LOSSES.[1]

Reading No.	Impressed volts E	Exciting current I	Field current on alternator.	Core loss. W	Frequency f	$\frac{\text{Watts}}{f}$	$\frac{E}{f} = C$
1	145.0	0.401	0.65	26.2	40	0.655	3.63
2	162.0	.403	"	29.8	45	.662	3.60
3	181.5	.405	"	33.9	50	.678	3.63
4	199.5	.407	"	37.4	55	.680	3.63
5	219.5	.410	"	41.9	60	.698	3.66
6	220.5	.412	"	42.1	60	.701	3.67
7	237.2	.415	"	45.8	65	.705	3.65
8	256.5	.417	"	49.3	70	.705	3.66
9	275.5	.420	"	54.0	75	.720	3.67
10	292.5	.421	"	58.3	80	.728	3.66

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[1] Core loss test with constant excitation.

DATA FOR SEPARATION OF LOSSES.[1]

Reading No.	Impressed volts.	Exciting Current.	Field current on alternator.	Frequency	Core loss + copper loss.	I^2R loss.	Core loss.
	E	I		f	watts	watts	watts
1	220.	0.260	0.458	80	36.2	0.006	36.2
2	"	.280	.500	75	37.2	.008	37.2
3	"	.310	.547	70	38.5	.010	38.5
4	"	.352	.600	65	39.8	.012	39.8
5	"	.412	.654	60	43.1	.017	43.1
6	"	.537	.730	55	44.8	.029	44.8
7	"	.774	.802	50	47.4	.060	47.3
8	"	.770	.817	50	47.7	.060	47.6
9	"	1.950	.943	45	53.3	.378	52.9
10	"	4.520	1.120	41	59.2	2.04	57.2

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[1] Core loss test with constant voltage impressed.

REGULATION CURVE DATA.

	(1)	(2)	(3)
Load in percent of full load.	25	100	150
Resistance of pri. (2 coils in series).	8.851	8.851	8.851
Full-load current in primary. $= 2.27^a$.			
Primary current.	.5675	2.27	3.405
I R drop in primary.	5.03	20.1	30.15
Normal voltage across pri. $= 2200.^v$			
% I R drop in primary.	0.229	0.914	1.371
Resistance of sec. (2 coils in series).	0.1089	0.1089	0.1089
Full load current in secondary $= 22.7^a$			
Secondary current.	5.675	22.7	34.05
I R drop in secondary.	0.618	2.47	3.71
Normal voltage across sec. $= 220.^v$			
% I R drop in secondary.	0.281	1.124	1.687
Total % I R drop in pri. + sec.	0.510	2.038	3.058
Impedance drop, I Z. (Volt-amp. curve.)	14.1	56.5	84.8
Watts I^2R. (short-circuit Amp.-watt curve).	6.0	94.0	205.0
Resistance drop, $IR = \frac{\text{watts}}{I}$.	10.57	41.4	60.2
$(IZ)^2$.	199.	3190.	7180.
$(IR)^2$.	111.7	1713.	3625.

	(1)	(2)	(3)
$(IX)^2 = (IZ)^2 - (IR)^2$.	87.3	1477.	3555.
IX.	9.34	38.4	59.6
Total % reactance drop $= \frac{IX}{2200}$.	0.424	1.745	2.71
Watts core loss on open circuit.	41.5	41.5	41.5
Exciting current, I_{ex}.	.411	.411	.411
Core loss energy current, $I_{he} = \frac{41.5}{220}$.	.1887	.1887	.1887
$I_{mag.} = \sqrt{(I_{ex})^2 - (I_{he})^2}$.	0.365	0.365	0.365
E.	100.	100.	100.
Inductance factor, $q = \frac{I_{mag.}}{I}$.	0.0643	0.0161	0.01073
qIX.	0.0272	0.0281	0.0291
$E + IR + qIX$.	100.54	102.07	103.09
$(E + IR + qIX)^2$.	10108.	10414.	10620.
$(E + IR + qIX)^2 + (IX)^2$.	10108.2	10417.0	10627.3
$E_0 = \sqrt{(E + IR + qIX)^2 + (IX)^2}$.	100.54	102.085	103.136
% Regulation.	0.54	2.085	3.136

EFFICIENCY CURVE DATA.

Load in percent of full load.	25.	100.	150.
Pri. full-load current = 2.27^a.			
Primary current.	.5675	2.27	3.405
Resistance of pri. (2 coils in series).	8.851	8.851	8.851
I^2R loss in primary.	2.85	45.6	102.7
Sec. full-load current = 22.7^a.			
Secondary current.	5.675	22.7	34.05
Resistance of sec. (2 coils in series).	0.1089	0.1089	0.1089
I^2R loss in secondary.	3.5	56.0	126.3
Total copper loss.	6.35	101.6	229.0
Core loss. (constant).	41.5	41.5	41.5
Total losses.	47.85	143.1	270.5
Output.	1250.	5000.	7500.
Input = Output + Total losses.	1297.9	5143.1	7770.5
% Efficiency.	96.3	97.3	96.5

THREE-PHASE CORE LOSS DATA.[1]

Reading No.	Voltage from line #2 to neutral. E	Voltage between lines #2 & #3. E	Current in line #2. I	Current in line #3. I	Core loss. W.
1	50.0	83.3	0.320	-----	6.8
2	60.0	99.5	.358	0.400	10.4
3	75.0	122.7	.420	.464	15.3
4	90.0	145.0	.495	.540	21.6
5	100.0	159.0	.547	.600	26.0
6	110.0	175.0	.630	.685	31.0
7	120.0	189.3	.724	.790	35.5
8	130.0	205.0	.853	.930	40.7
9	148.0	230.0	1.220	1.320	51.3
10	164.0	251.5	1.733	1.833	62.3
11	169.0	258.0	1.965	2.040	66.5
12	180.0	273.0	2.680	2.790	74.7
13	192.0	288.5	4.200	4.250	85.5
14	210.0	310.5	8.800	8.500	
15	225.0	329.1	14.25	13.55	
16	231.0	337.5	17.30	16.90	
17	237.0	346.5	20.10	19.60	
18	240.0	351.0	21.60	21.20	

[1] Connections :- Y-Primary and Y-Secondary.

THREE - PHASE CORE LOSS DATA.[1]

Reading No.	Voltage, line #2 to neutral. (primary) E	Voltage between lines #2 + #3 (primary) E	Voltage across #2 (secondary) E	(a) Voltage across open delta. (secondary). E	Triple-f. e.m.f across opening in delta. E	Current in line #2. I	Current in line #3. I	Core loss. W
1	60.0	97.8	60.0	69.0	49.0	0.360	0.410	14.6
2	72.0	117.0	72.0	85.0	63.0	.415	.460	19.4
3	84.6	135.6	84.6	102.5	80.0	.480	.530	24.2
4	96.0	153.6	96.0	121.0	107.0	.565	.610	30.0
5	108.0	170.4	108.0	139.0	127.0	.670	.730	35.9
6	110.0	174.0	110.0	142.0	130.0	.690	.755	36.4
7	120.0	193.2	120.0	157.0	148.5	.815	.890	41.5
8	134.0	205.8	132.0	168.0	163.5	.950	1.033	46.5
9	144.0	224.4	144.0	185.5	186.0	1.200	1.29	53.4
10	156.0	240.0	156.0	205.0	211.0	1.58	1.70	62.7
11	168.6	267.6	168.6	223.0	236.5	2.22	2.31	72.5
12	181.2	273.6	181.2	245.0	268.0	3.34	3.39	82.5
13	188.4	282.0	188.4	258.0	287.5	4.55	4.55	89.4
14	204.6	301.2	204.6	286.0	330.0	8.20	8.30	98.0
15	217.2	316.8	217.2	306.0	358.0	12.50	12.50	121.0
16	228.0	331.8	228.0	322.0	380.0	17.90	17.50	144.0
17	231.6	334.8	231.6	330.0	389.0	20.2	19.8	
18	234.6	342.0	234.6	334.0	395.0	22.6	22.0	

[1] Connections :- Y-Primary and Open △ Secondary.

THREE-PHASE CORE LOSS DATA.[1]

Reading No.	Voltage, line #2 to neutral (primary) E	Voltage between lines #2 + #3 E	Voltage, across # 2 (secondary) E	Current in line #2. I	Current in line #3. I	Triple frequency current. I	Core loss. W
1	70.0	121.5	70.0	0.433	0.485	0.060	19.4
2	80.0	139.0	80.0	.495	.550	.120	24.2
3	90.0	156.0	90.0	.579	.632	.150	31.7
4	100.5	174.0	100.5	.683	.755	.205	35.8
5	105.0	182.0	105.0	.743	.828	.240	38.0
6	110.0	190.7	110.0	.826	.913	.285	40.7
7	115.0	199.0	115.0	.915	1.013	.340	43.6
8	125.0	217.0	125.0	1.19	1.320	.494	49.0
9	134.5	234.0	134.5	1.66	1.815	.805	51.4
10	140.0	243.0	140.0	2.18	2.26	1.130	46.6
11	145.0	252.0	145.0	2.97	3.02	1.68	38.6
12	150.5	262.0	150.5	3.89	3.88	2.44	30.3
13	153.0	267.0	153.0	4.68	4.64	2.86	22.6
14	155.0	269.5	155.0	5.10	5.2	3.24	9.7
15	162.5	283.0	162.5	7.8	7.2	4.75	-9.7
16	170.3	295.0	170.0	10.6	9.2	6.90	-22.3
17	174.0	303.0	174.6	13.9	12.9	8.3	-29.0
18	180.6	315.0	180.6	17.6	16.7	10.6	
19	183.0	320.4	183.0	20.2	19.0	11.9	
20	188.4	329.7	188.4	24.8	23.5	14.2	

[1] Connections :- Y-Primary and Closed Δ Secondary.

THREE-PHASE CORE LOSS DATA.[1]

Reading No.	Voltage, line #2 to neutral. E	Voltage between lines #2+#3 E	Current in line #2 I	Current in line #3. I	Triple frequency current. I	Core loss of three transformers W	Core loss per transformer. W
1	60.0	103.8	0.400	0.430		46.6	15.53
2	70.2	121.7	.450	.490	0.05	60.3	20.10
3	80.0	138.7	.510	.560	.12	75.4	25.13
4	89.7	165.0	.570	.635	.15	91.7	30.54
5	99.8	173.0	.670	.757	.21	109.3	36.44
6	105.0	182.0	.745	.832	.252	119.0	39.68
7	110.0	190.5	.810	.900	.28	134.0	44.64
8	120.5	209.0	1.040	1.150	.411	169.6	56.50
9	130.6	225.5	1.400	1.510	.65	201.0	67.00
10	134.5	235.0	1.71	1.835	.862	228.5	76.00
11	144.0	251.0	2.84	2.90	1.61	249.0	83.00

[1] Obtained by two wattmeter method.

Connections :- Y-Primary and Closed-Δ Secondary.

HYSTERESIS LOOP DATA.[1]

Reading No.	Exciting current. $I_{d.c.}$	Galvanometer deflection m.m.	Deflection proportional to $\beta_{max.}$	% $\beta_{max.}$	% $I_{max.}$
(1)	(2)	(3)	(4)	(5)	(6)
1	+0.583	---	2913	100.0	100.0
2	+ .533	22	2891	99.2	91.4
3	+ .303	183	2708	93.0	52.0
4	+ .179	198	2510	86.1	30.65
5	+ .048	354	2156	73.9	8.22
6	0.0	191	1965	67.4	0.0
7	- .031	154	1811	62.1	- 5.31
8	- .061	245	1566	53.6	- 10.47
9	- .101	353	1213	41.6	- 17.3
10	- .125	402	811	27.8	- 21.4
11	- .135	231	580	19.9	- 23.1
12	- .148	284	296	10.2	- 25.4
13	- .159	279	17	.583	- 27.2
14	- .178	415	-398	-13.7	- 30.5
15	- .199	373	- 771	- 26.5	- 34.1
16	- .222	380	-1151	- 39.5	- 38.0
17	- .240	214	-1365	- 46.8	- 41.1
18	- .270	282	-1647	- 56.5	- 46.2
19	- .304	258	-1905	- 65.4	- 52.0
20	- .349	250	-2155	- 73.9	- 59.8
21	- .389	185	-2340	- 80.2	- 66.6
22	- .500	370	-2710	- 93.0	- 85.6
23	- .583	203 5826	-2913	-100.0	-100.0
24	- .533	23			

(1)	(2)	(3)	(4)	(5)	(6)
25	- .302	187			
26	- .179	201			
27	- .048	364			
28	0.0	196			
29	+ .031	159			
30	+ .068	260			
31	+ .090	214			
32	+ .116	370			
33	+ .125	185			
34	+ .135	247			
35	+ .148	300			
36	+ .160	300			
37	+ .179	431			
38	+ .199	373			
39	+ .221	360			
40	+ .240	202			
41	+ .270	264			
42	+ .303	241			
43	+ .348	230			
44	+ .389	172			
45	+ .500	342			
46	+ .583	189			
		5818			

Total descending = 5826　　5826 ÷ 2 = 2913

" ascending = 5818　　5818 ÷ 2 = 2909

Diff. = 4 m.m.

1 At normal excitation.

HYSTERESIS LOOP DATA.[1]

Reading No.	Exciting current $I_{d.c.}$	Galv. defl. m.m.	Defl. prop. to $\beta_{max.}$	% $\beta_{max.}$	% $I_{max.}$	% $\beta_{max.}$ in terms of normal.	% $I_{max.}$ in terms of normal.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	+33.15	---	5717	100.0	99.2	196.0	5678.0
2	+27.40	110	5607	98.0	82.0	195.5	4695.0
3	+21.80	148	5459	95.4	65.2	187.3	3736.0
4	+16.10	196	5263	92.0	48.2	180.7	2760.0
5	+12.92	137	5126	89.6	38.5	176.0	2213.0
6	+10.04	146	4980	87.2	30.05	171.0	1720.0
7	+ 9.86	20	4960	86.7	29.5	170.4	1690.0
8	+ 8.42	85	4875	85.25	25.2	167.3	1443.0
9	+ 4.25	295	4580	80.2	12.73	157.2	727.8
10	+ 2.15	213	4367	76.3	6.43	149.8	368.4
11	+ 1.35	138	4229	74.0	4.04	145.2	231.2
12	+ 1.345	2	4227	73.9	4.025	145.0	230.7
13	+ .657	300	3927	68.7	1.97	134.8	112.6
14	+ .305	456	3471	60.7	.913	119.0	52.25
15	+ .237	75	3396	59.4	.710	116.5	40.6
16	+ .121	262	3134	54.8	.362	107.5	20.74
17	+ .022	344	2790	48.8	.066	95.8	3.77
18	0.0	92 3019	2698	47.2	0.0	92.6	0.0
19	- .064	373	2325	40.7	- .192	79.7	- 10.97
20	- .106	387	1938	33.9	- .317	66.5	- 18.16
21	- .128	314	1624	28.4	- .383	55.7	- 21.92
22	- .137	200	1424	24.9	- .410	48.8	- 23.48
23	- .150	301	1123	19.64	- .449	38.6	- 25.7
24	- .161	360	763	13.34	- .482	26.2	- 27.6

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
25	- .174	375	388	6.78	- .521	13.3	- 29.8
26	- .189	403	15	.26	- .566	.56	- 32.4
27	- .210	484	- 499	- 8.72	- .628	- 17.15	- 36.0
28	- .227	356	- 855	-14.96	- .680	- 29.4	- 38.9
29	- .237	150	-1005	-17.58	- .710	- 34.5	- 40.6
30	- .260	320	-1325	-23.15	- .778	- 45.5	- 44.6
31	- .288	270	-1595	-27.9	- .862	- 54.7	- 49.3
32	- .347	500	-2095	-36.6	- 1.04	- 71.9	- 59.5
33	- .431	455	-2550	-44.6	- 1.29	- 87.5	- 73.8
34	- .521	318	-2868	-50.2	- 1.56	- 98.4	- 89.2
35	- .693	404	-3272	-57.2	- 2.08	-112.3	- 118.8
36	- .954	361	-3633	-63.5	- 2.86	-124.7	- 163.5
37	- 1.360	303	-3936	-68.7	- 4.07	-135.0	- 233.0
38	- 1.365	5	-3941	-68.9	- 4.08	-135.3	- 234.0
39	- 2.175	268	-4209	-73.5	- 6.51	-144.3	- 373.0
40	- 4.34	305	-4514	-78.9	-13.0	-155.0	- 744.0
41	- 9.88	420	-4934	-86.2	-29.6	-169.3	-1695.0
42	-10.05	10	-4944	-86.3	-30.1	-169.6	-1722.0
43	-12.61	140	-5084	-88.9	-37.8	-174.6	-2160.0
44	-17.00	195	-5279	-92.3	-50.9	-181.2	-2910.0
45	-21.90	165	-5444	-95.1	-65.5	-186.8	-3754.0
46	-25.40	100	-5544	-97.0	-76.0	-190.2	-4350.0
47	-33.40	173 8415 3019 11434	-5717	-100.0	-100.0	-196.0	-5724.0
48	-27.60	111	-5606				
49	-20.50	194	-5412				

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
50	-16.60	134	-5278				
51	-13.20	145	-5133				
52	-10.04	160	-4973				
53	- 9.91	9	-4964				
54	- 8.43	83	-4881				
55	- 4.25	297	-4584				
56	- 2.145	212	-4372				
57	- 1.350	136	-4236				
58	- 1.345	2	-4234				
59	- .657	299	-3935				
60	- .370	306	-3629				
61	- .236	219	-3410				
62	- .121	255	-3155				
63	- .022	341	-2814				
64	0.0	91	-2723				
65	+ .064	374	-2349				
66	+ .105	385	-1964				
67	+ .127	310	-1654				
68	+ .137	196	-1458				
69	+ .149	298	-1160				
70	+ .160	357	- 803				
71	+ .173	372	- 431				
72	+ .189	403	- 28				
73	+ .208	484	456				
74	+ .227	359	815				
75	+ .235	151	966				

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
76	+ .259	328	1294				
77	+ .287	314	1608				
78	+ .343	499	2057				
79	+ .431	470	2527				
80	+ .519	318	2845				
81	+ .624	280	3125				
82	+ .734	219	3344				
83	+ .952	295	3639				
84	+ 1.358	310	3949				
85	+ 1.360	5	3954				
86	+ 2.175	271	4225				
87	+ 4.340	305	4530				
88	+ 9.970	431	4961				
89	+10.00	2	4963				
90	+11.47	82	5045				
91	+16.00	217	5262				
92	+22.45	223	5485				
93	+26.25	107	5592				
94	+33.05	145	5737				
		11454					

Total descending = 11534 11434 ÷2 = 5717

" ascending = 11454 11454 ÷ 2 = 5727

Diff. 40

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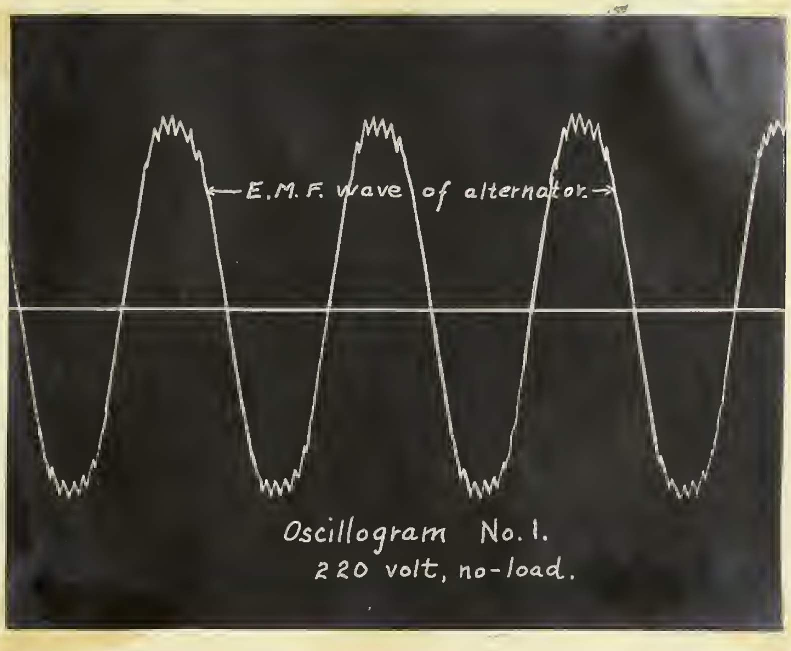
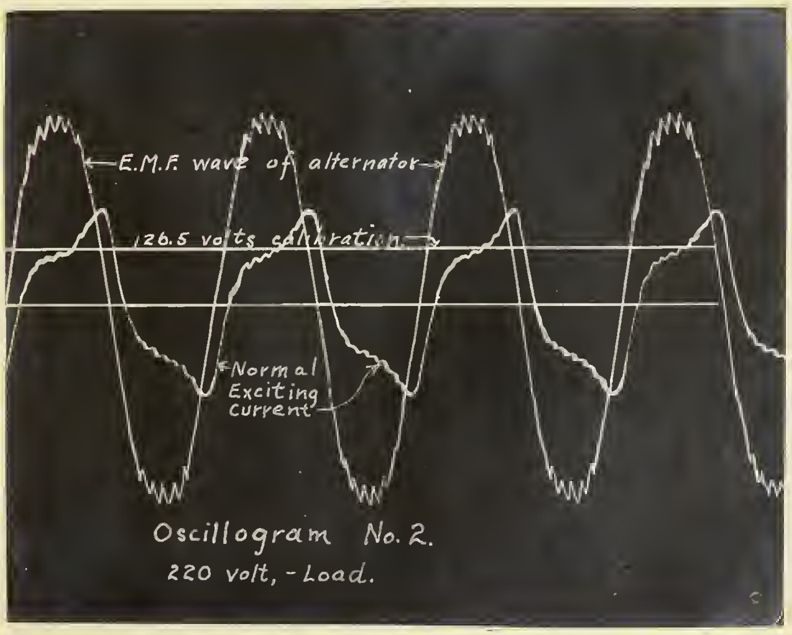
1 With full-load current as exciting current.

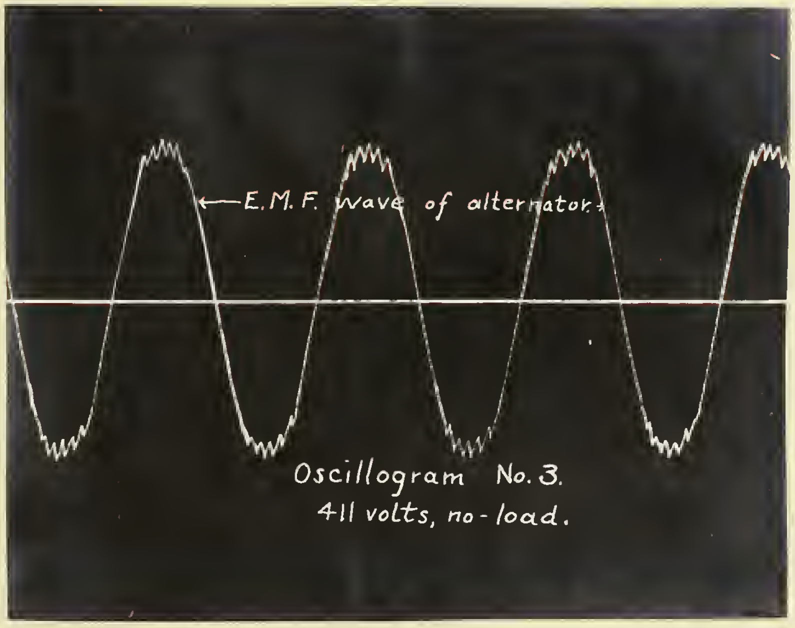


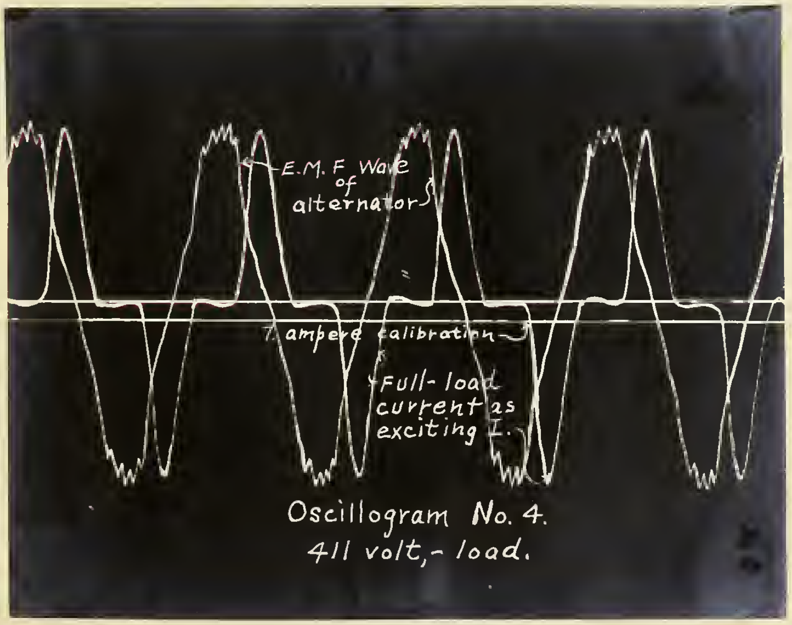
Oscillogram No. 1.
220 volt, no-load.



Oscillogram No. 2.
220 volt, - Load.

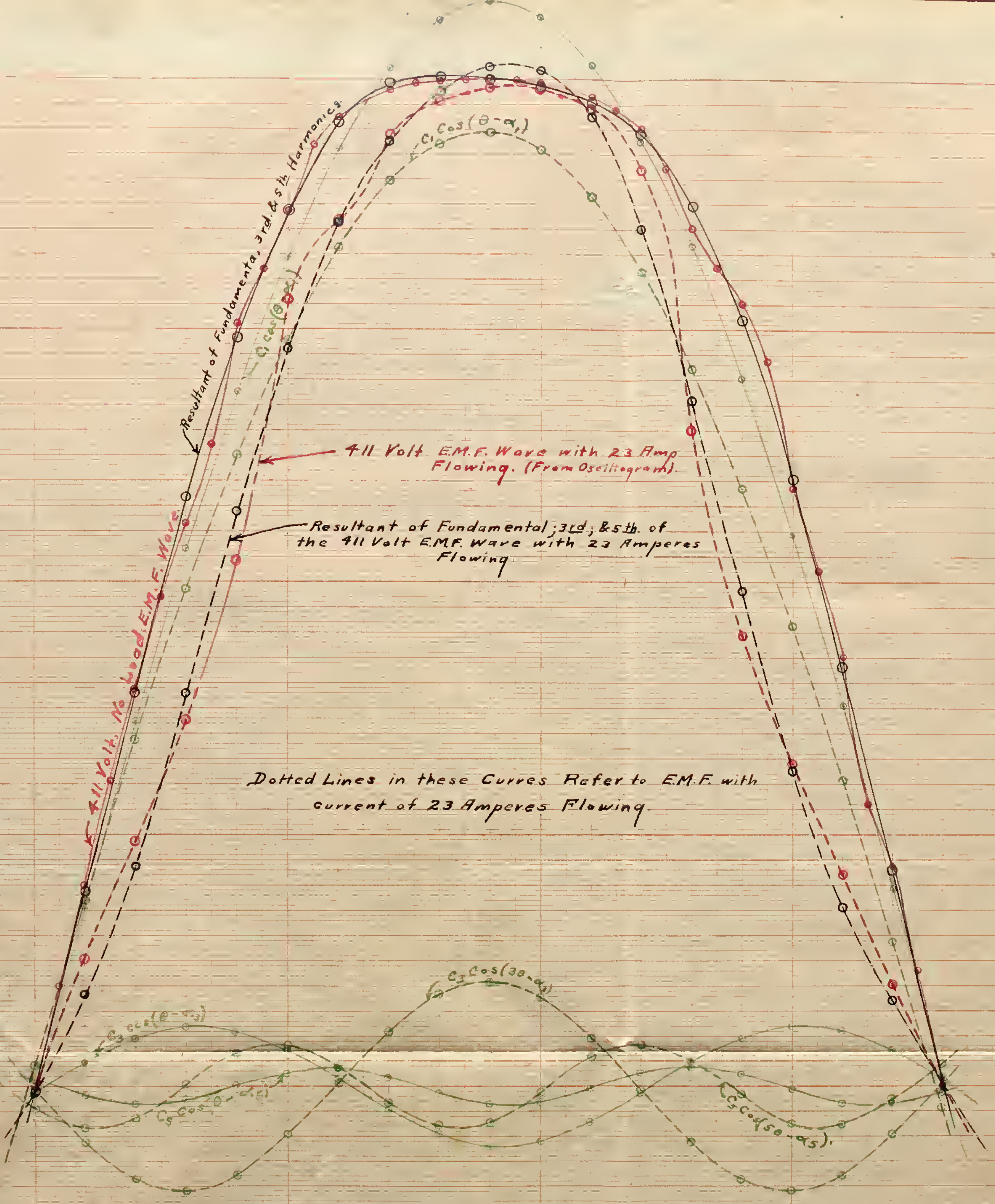
Curve Sheet No. 1.



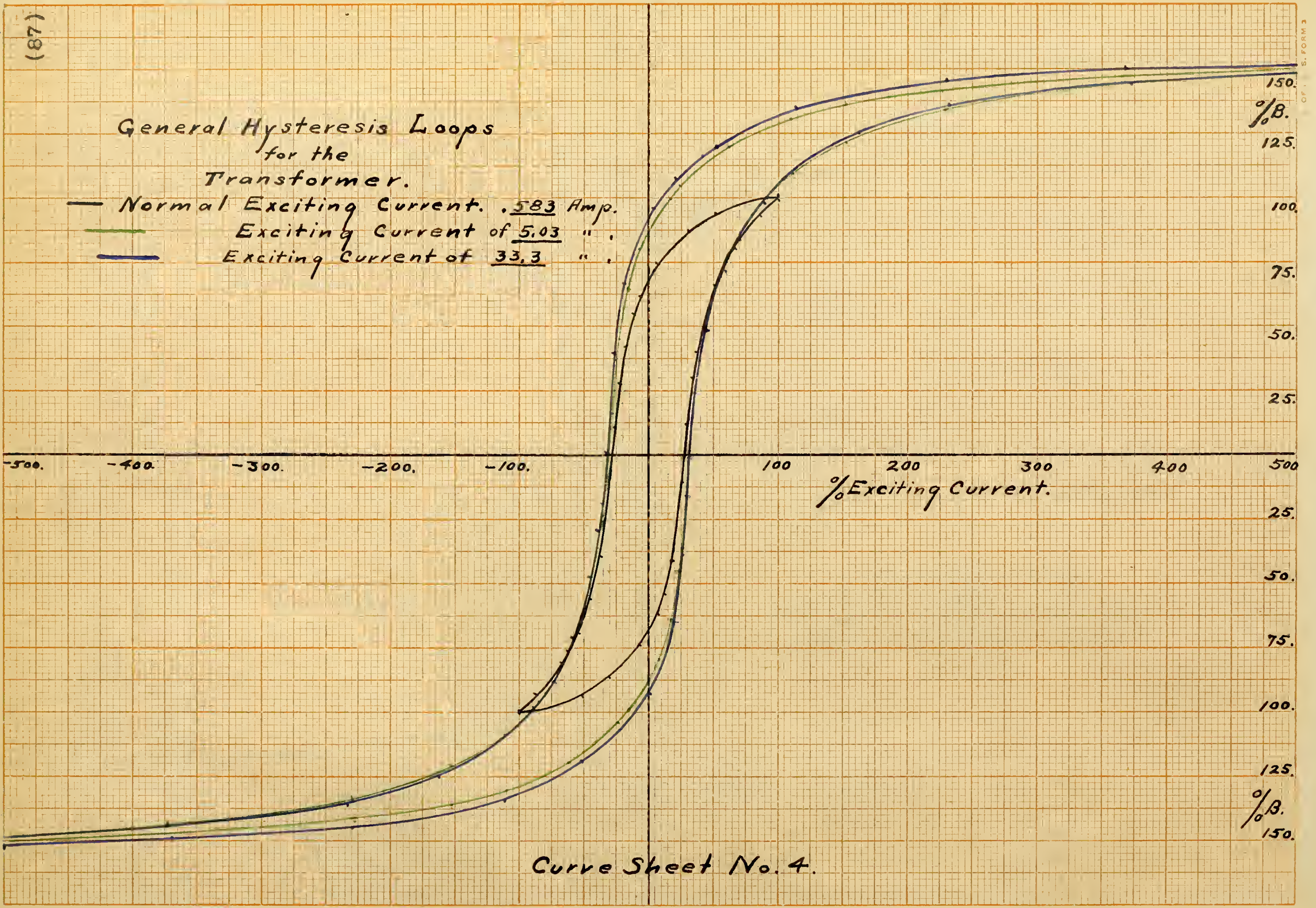


Curve Sheet No. 2.

Curve Sheet No. 3.

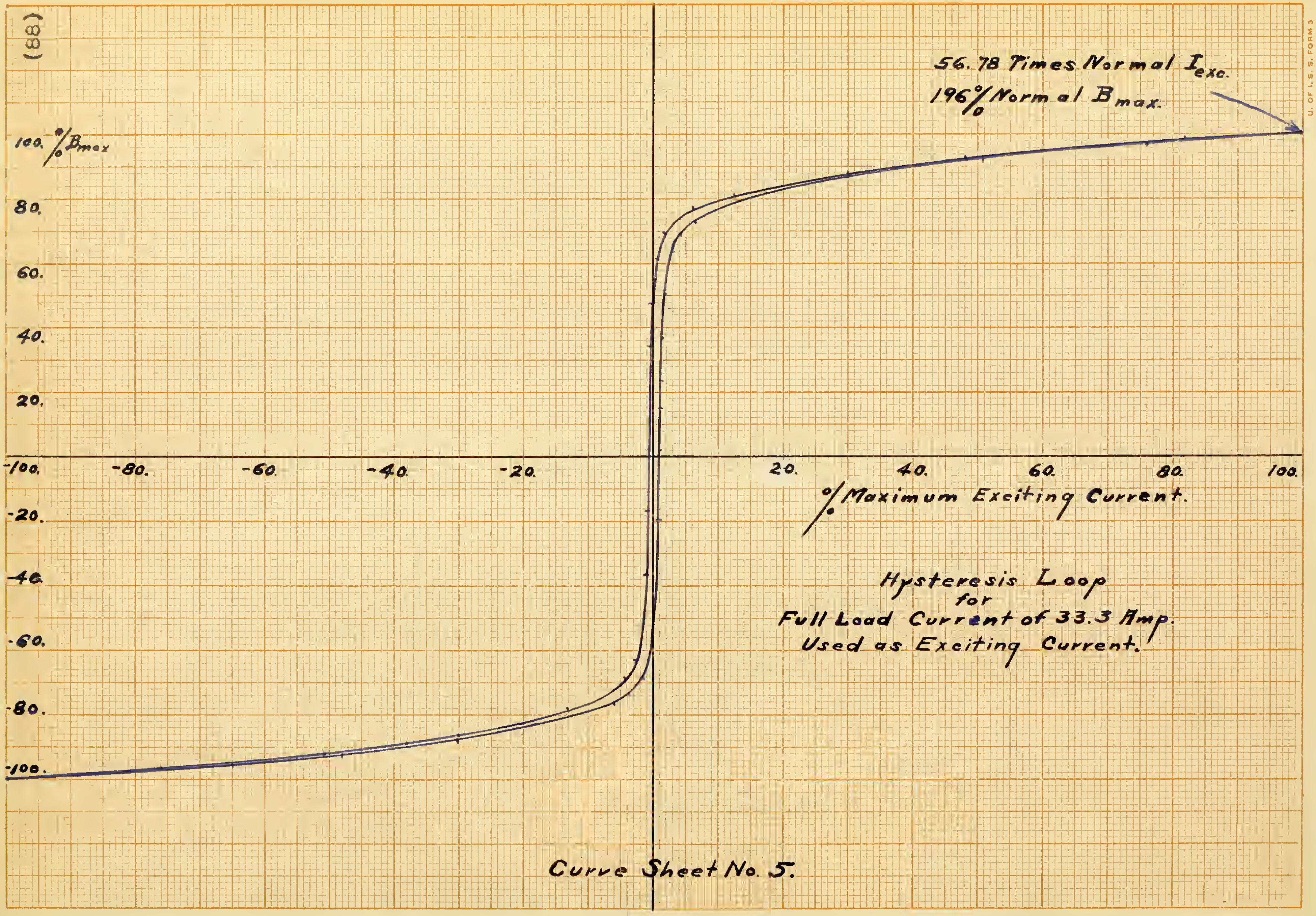


Dotted Lines in these Curves Refer to E.M.F. with current of 23 Amperes Flowing.



General Hysteresis Loops
for the
Transformer.

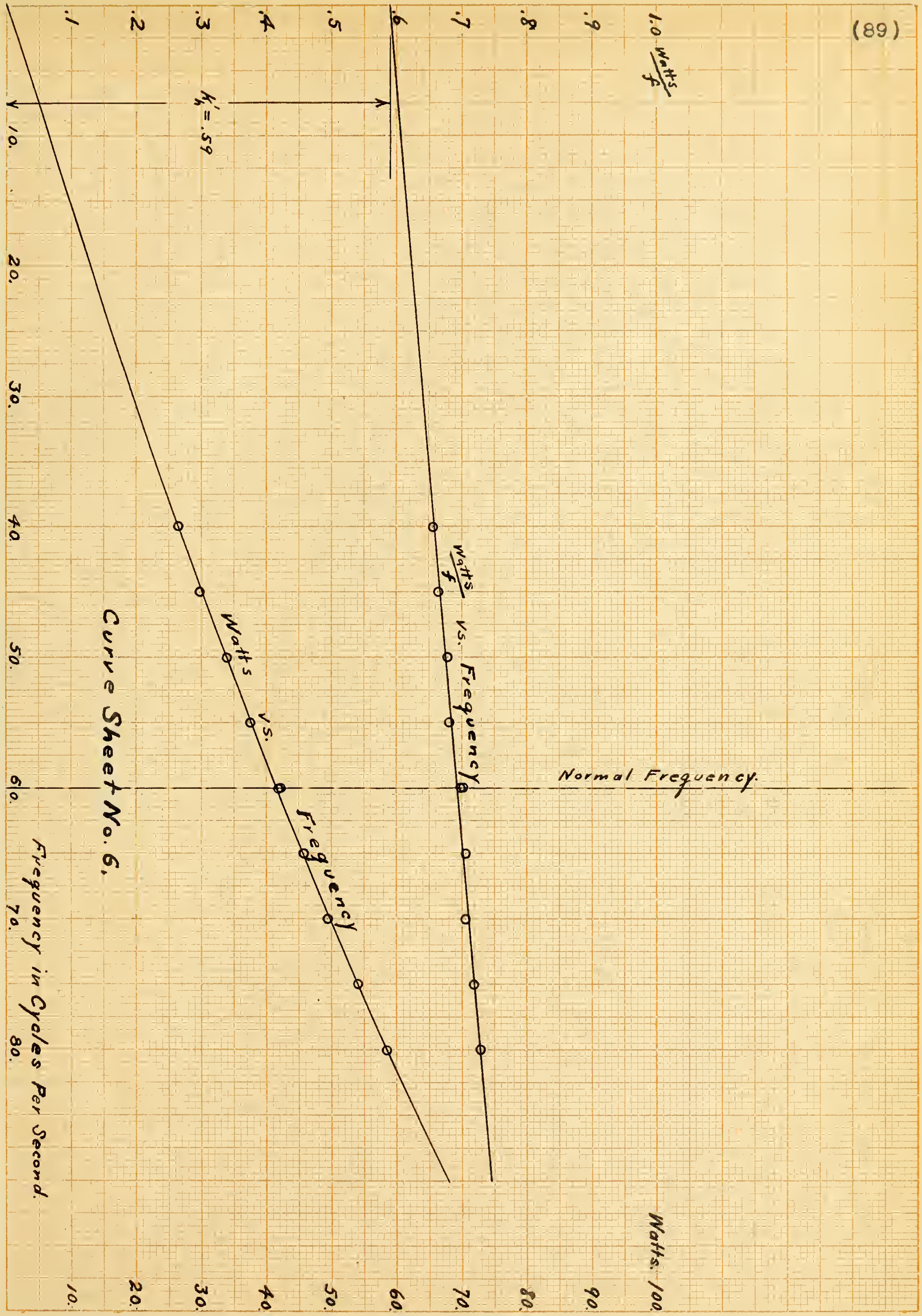
Curve Sheet No. 4.



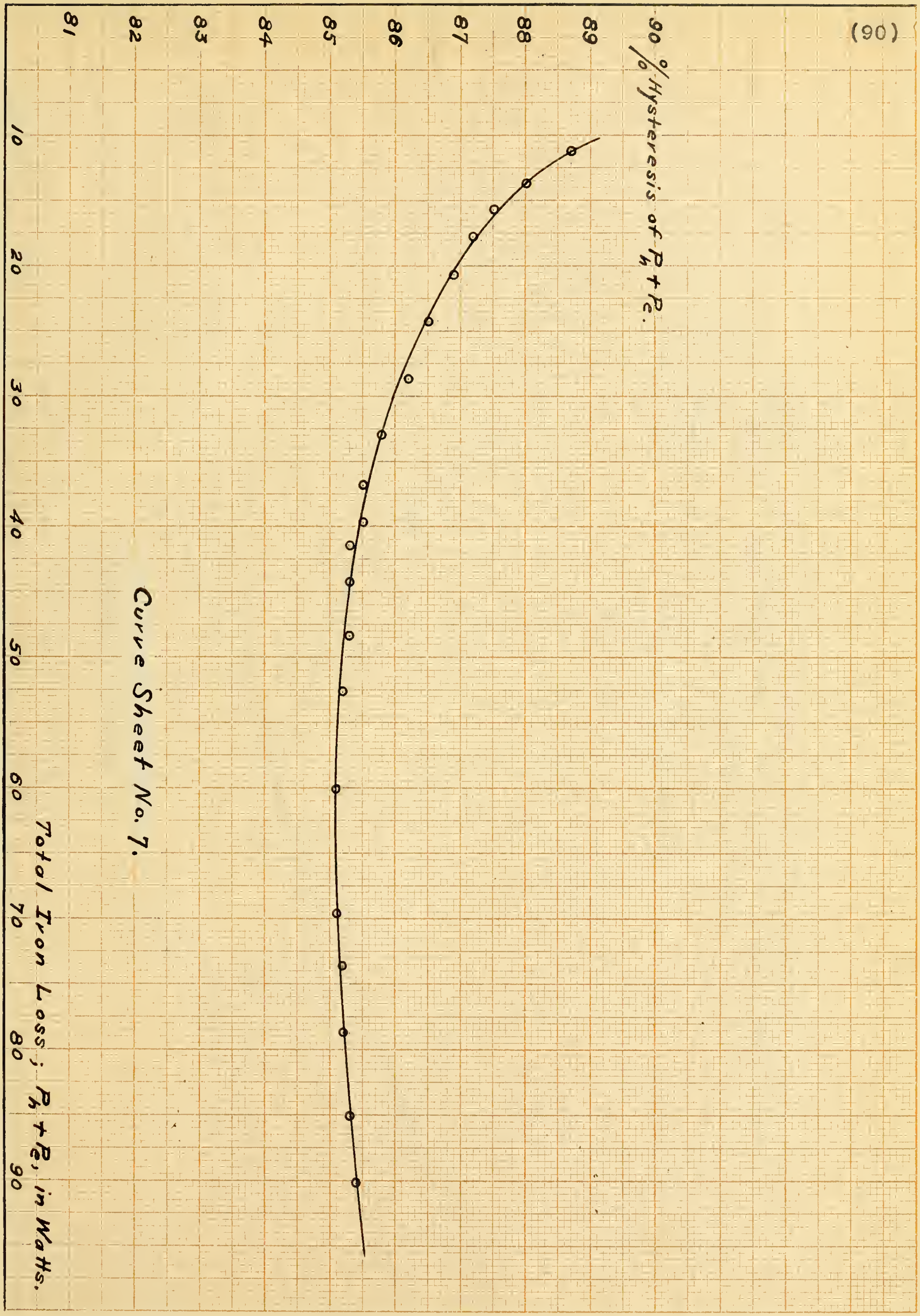
Hysteresis Loop
for
Full Load Current of 33.3 Amp.
Used as Exciting Current.

Curve Sheet No. 5.

U. OF I. S. S. FORM 3

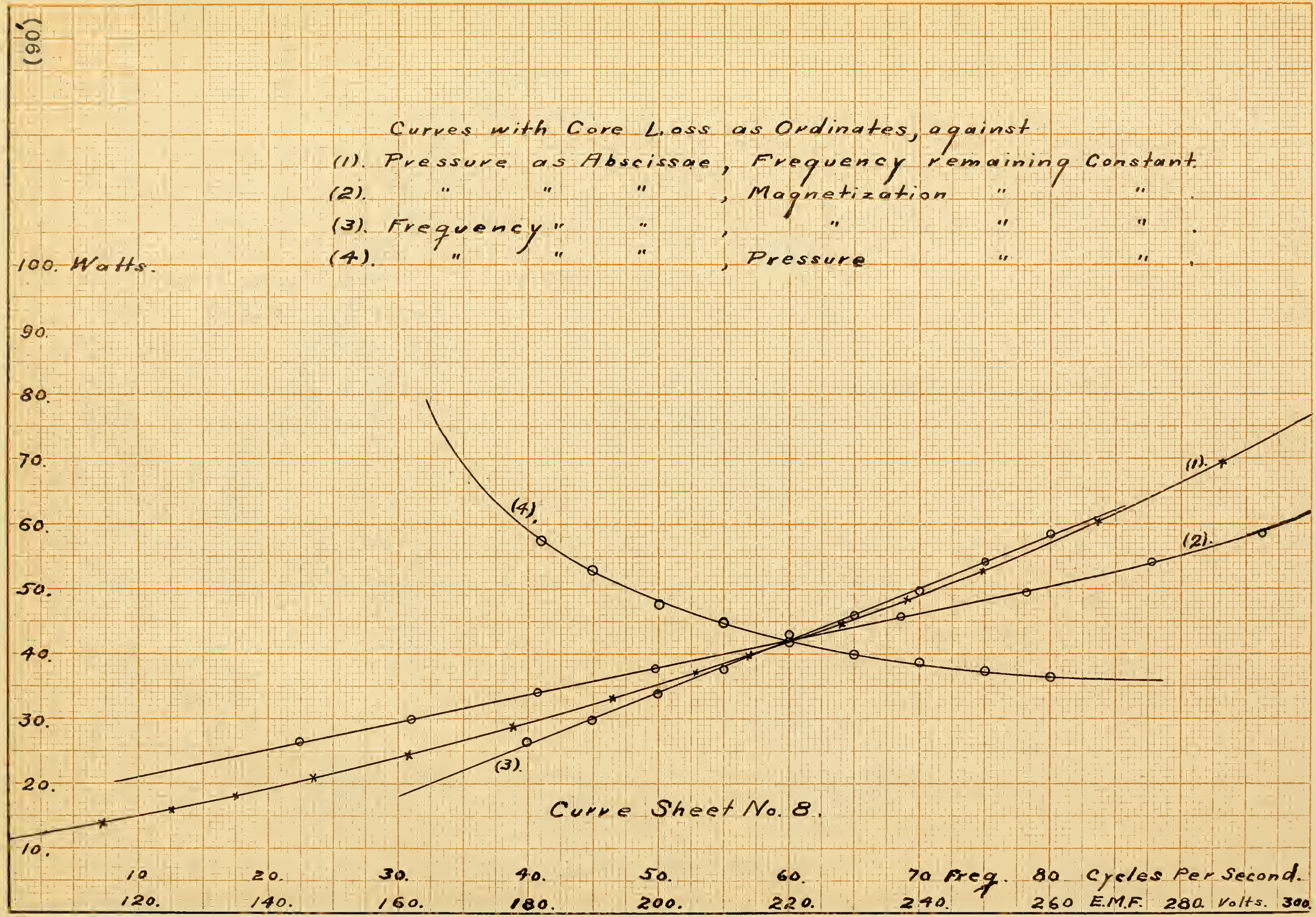


Curve Sheet No. 6.

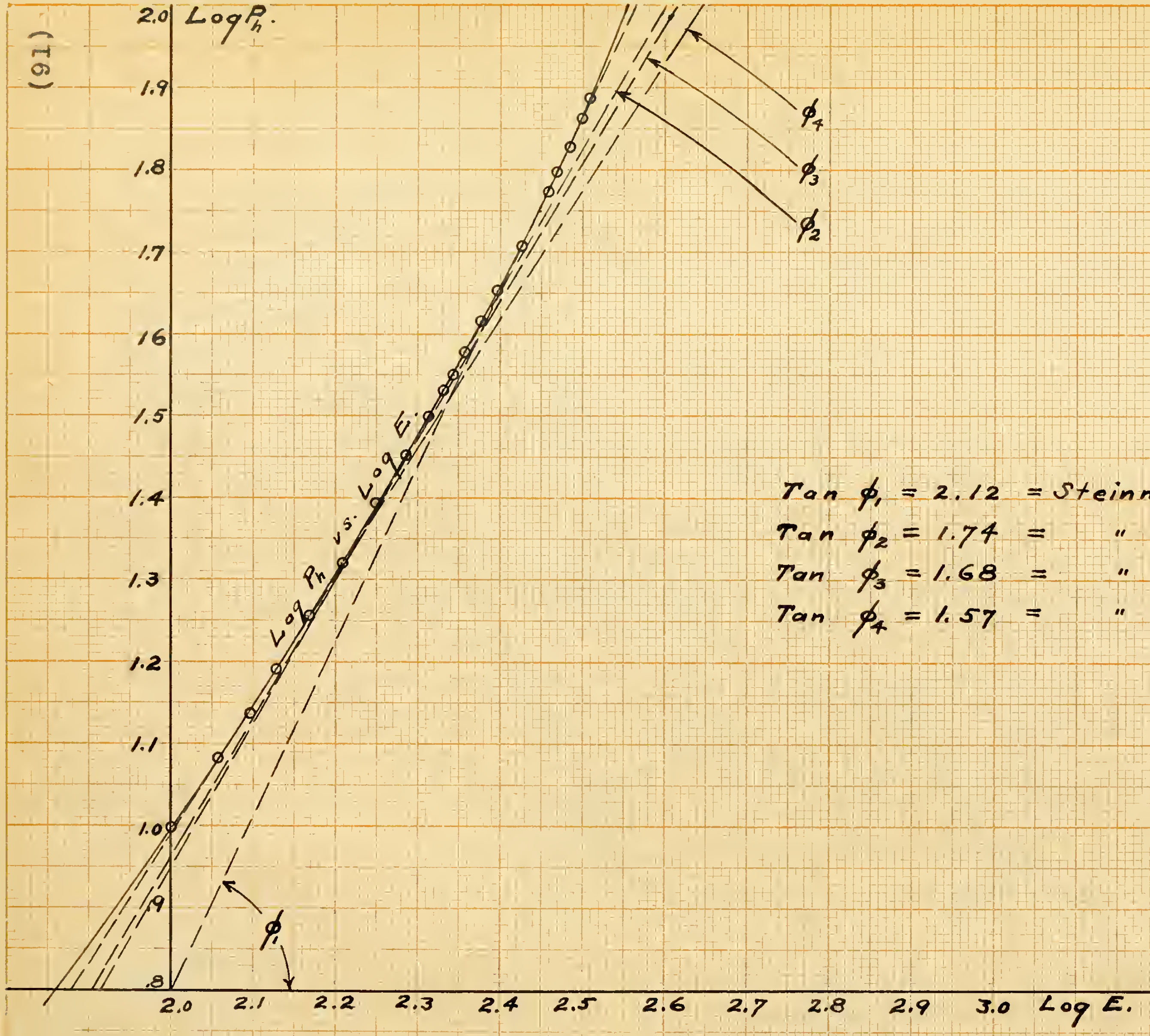


Curve Sheet No. 7.

U. OF I. E. S. FORM 3



U. OF I. S. S. FORM 3



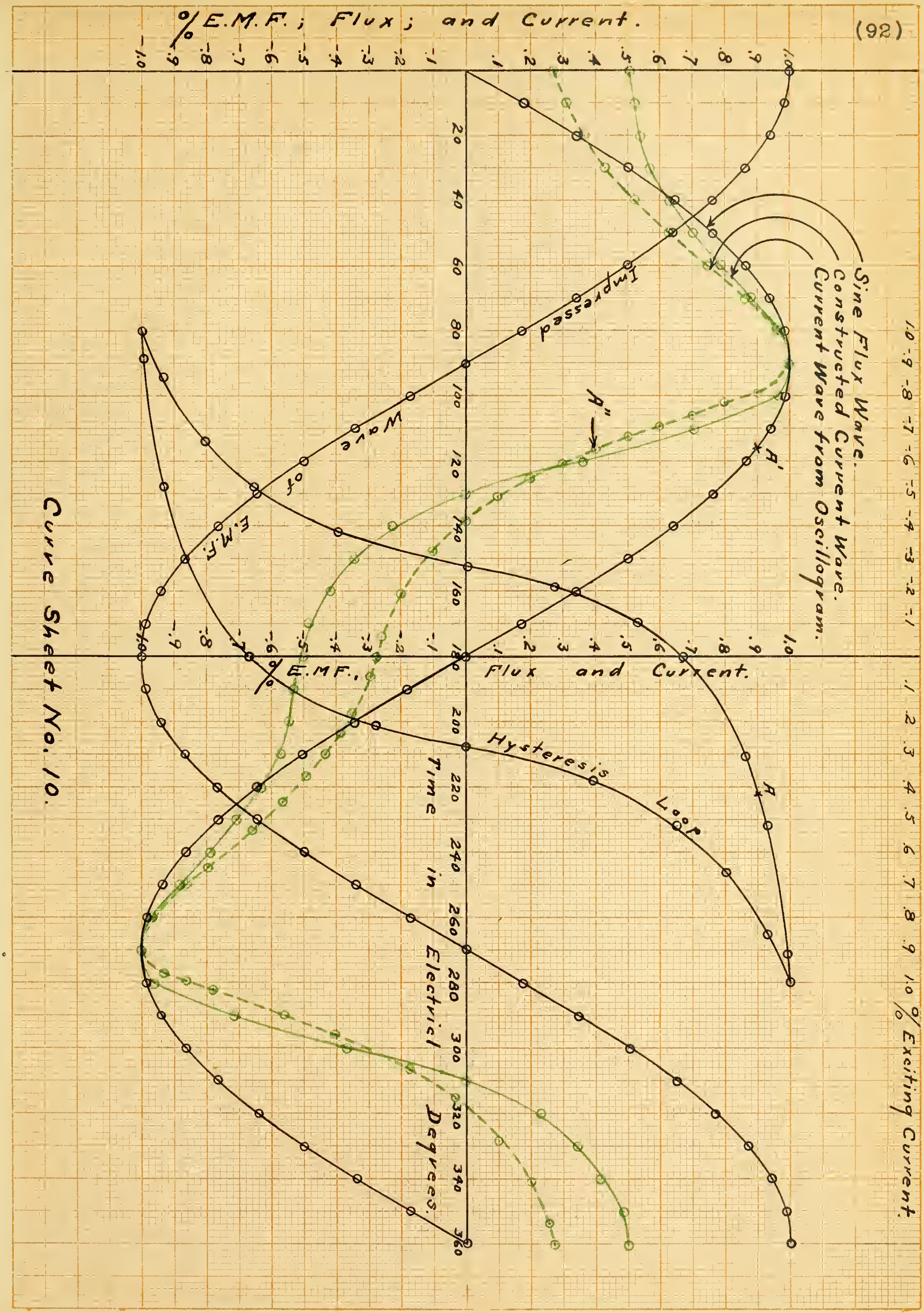
Tan ϕ_1 = 2.12 = Steinmetz' Exponent.

Tan ϕ_2 = 1.74 = " " .

Tan ϕ_3 = 1.68 = " " .

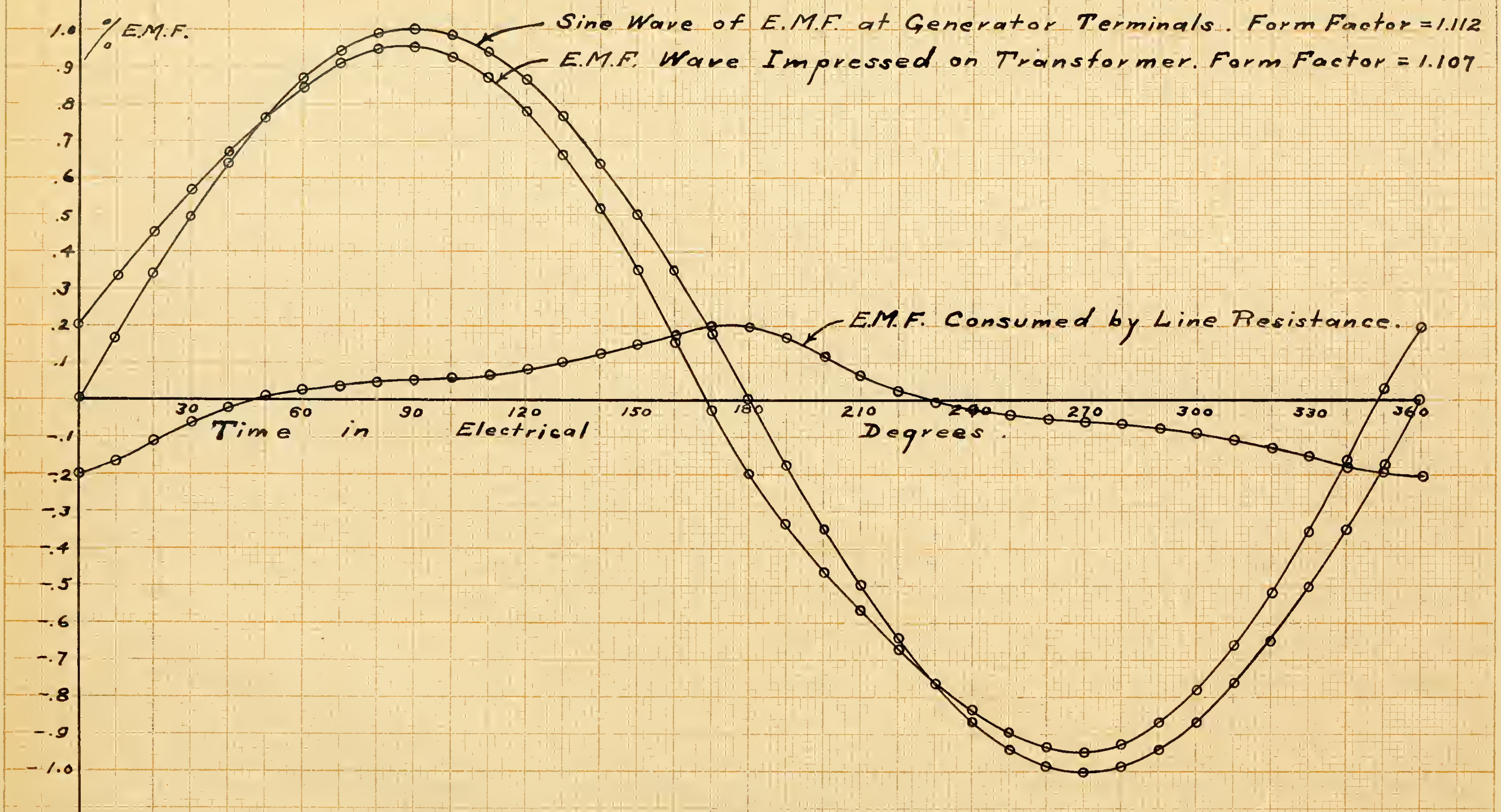
Tan ϕ_4 = 1.57 = " " .

Curve Sheet No. 9.



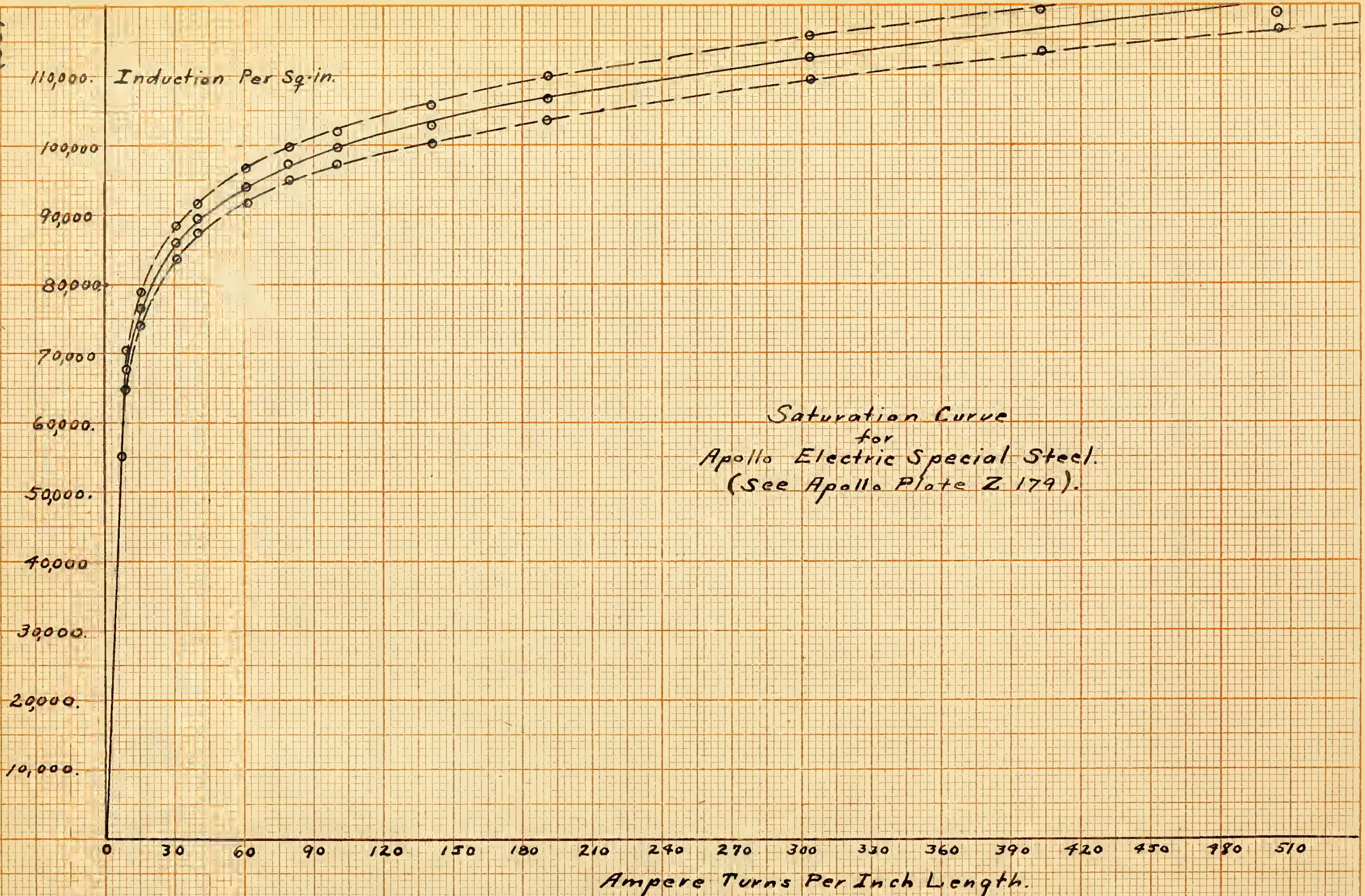
Curve Sheet No. 10.

U. OF I. S. S. FORM 3



Curve Sheet No. 11.

U. OF I. S. S. FORM 3

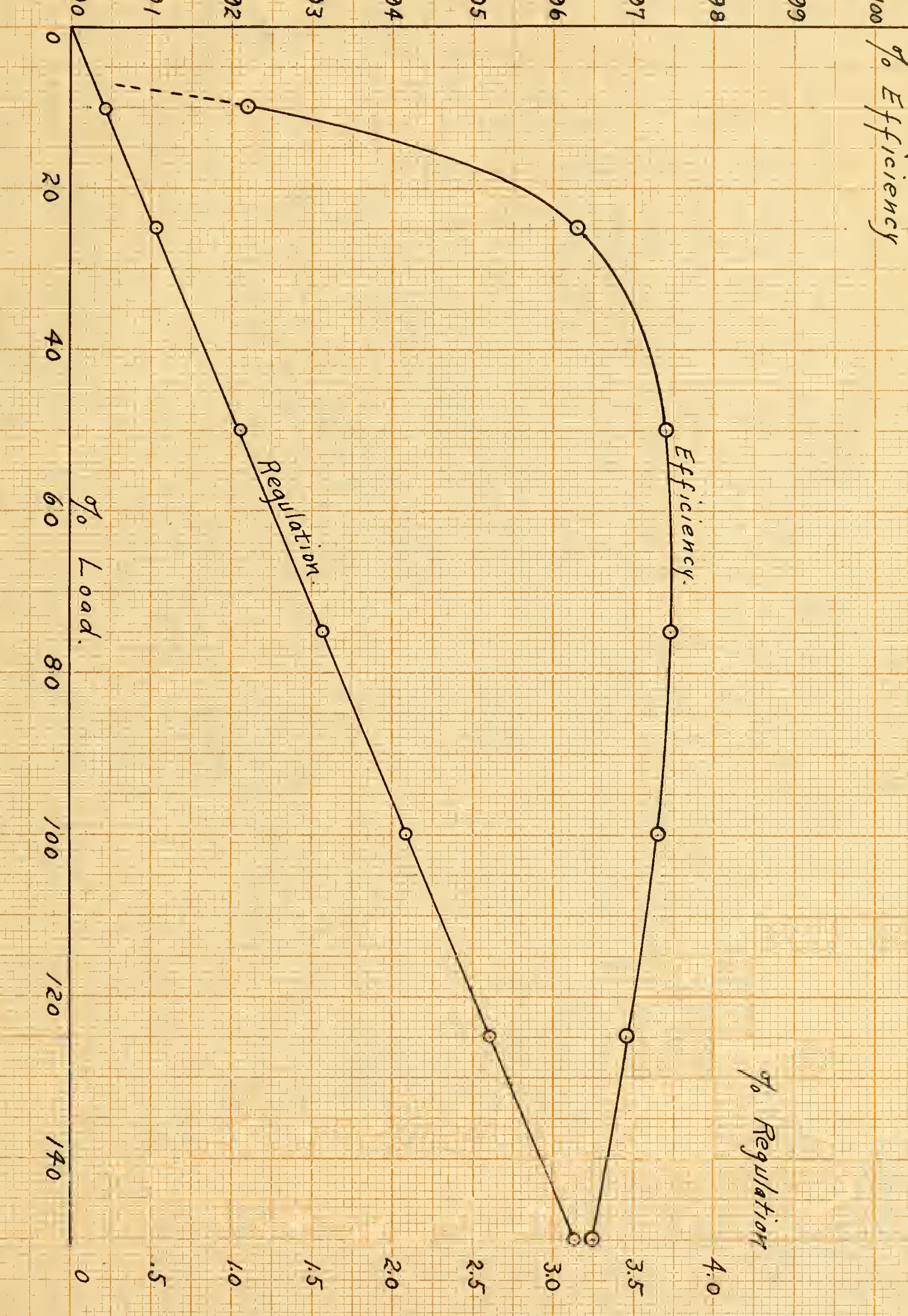


Saturation Curve
for
Apollo Electric Special Steel.
(See Apollo Plate Z 179).

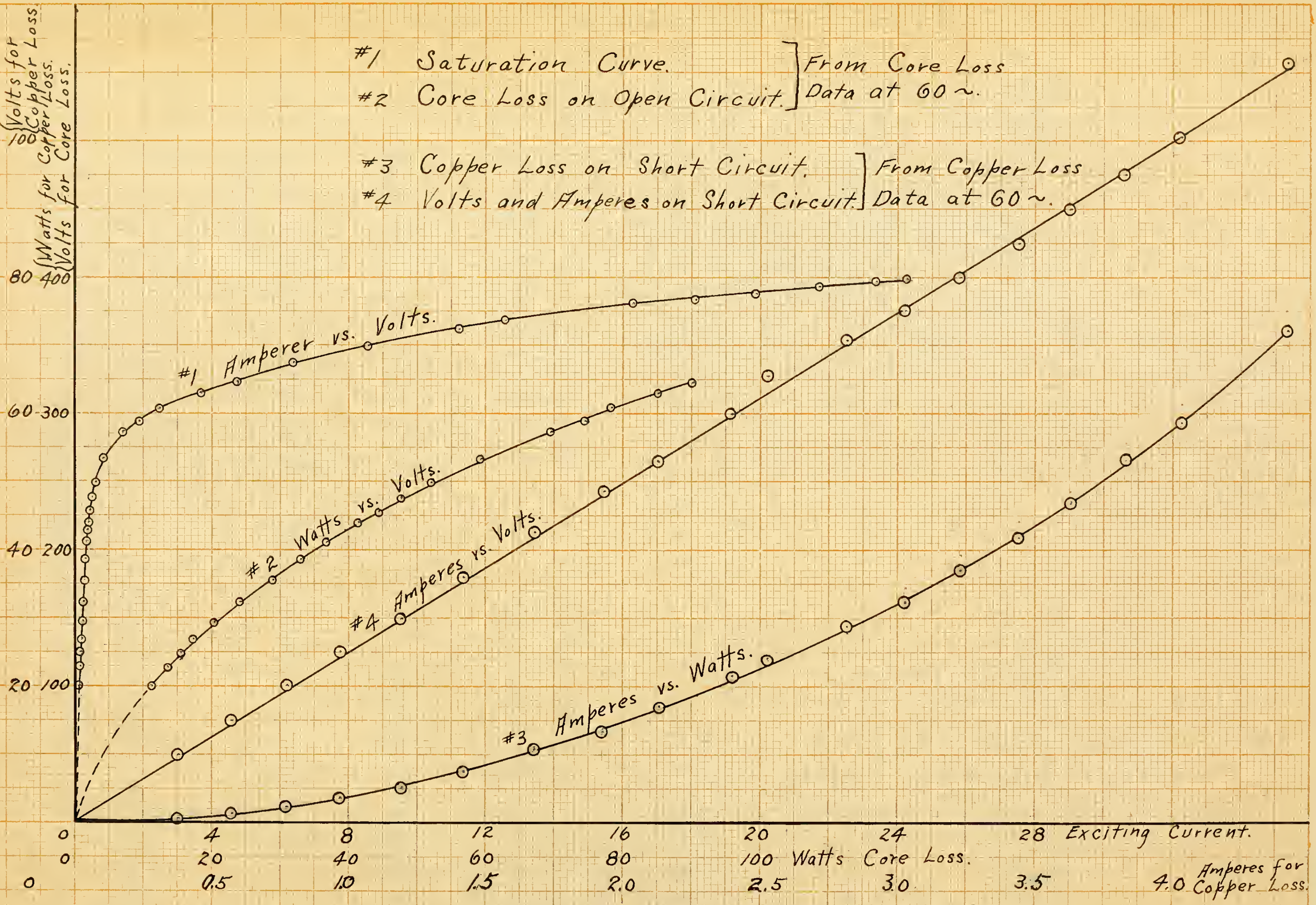
Curve Sheet No. 12.

Univ. of Illinois.
Oct. 11, 1910. Wooten.

U. OF I. S. S. FORM 2



Curve Sheet No. 13.



Curre Sheet No. 14.

U. OF I. S. S. FORM 3

References.

Definition of hysteresis:	"Electrical Measurements", Carhart & Patterson, page 299.
Ballistic method of determination of hysteresis loop:	"Testing of Electro-Magnetic Machinery", Swenson & Frankenfield, Vol.II, page 91.
Hysteresis losses:	"Electricity and Magnetism", Gerard.
Work of hysteresis, and meaning of the hysteresis loop:	"Electricity and Magnetism", Gerard, pages 71, 94 and 281.
Analysis of wave form:	"Testing of Electro-Magnetic Machinery", Swenson & Frankenfield, Vol.II, page 219.
Regulation:	"Testing of Electro-Magnetic Machinery", Swenson & Frankenfield, pages 83 to 88.
Magnetometric method of determination of μ :	"Electrical Measurements", Carhart & Patterson, pages 295 to 303.

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